Here are hints on PS4. In problem 1, what is the breakdown point of a linear combination of order statistics? A hint is that it's equal to the smallest breakdown point of any of the order statistics appearing in the linear combination. If none of the individual order statistics breaks down (goes to infinity) then the linear combination won't. If one of them does break down, then the combination will, just as the sample mean breaks down if any one of the observations goes to infinity.

About problem 2: as shown in class, for odd sample size $n = 2k + 1$, the sum of $|X_i - \theta|$ is minimized with respect to θ when θ is the sample median $X_{(k+1)}$. You can differentiate the sum except where θ equals one of the X_i, and the sum is continuous also at those points. The derivative is negative for θ smaller than the median and positive for θ larger than the median, so the sample median gives a unique minimum for n odd.

For even sample size $n = 2k$, you can check similarly that the derivative is negative for $\theta < X_{(k)}$ and positive for $\theta > X_{(k+1)}$, except at finitely many observations X_i. You can guess from the problem statement what the value of the derivative should be between $X_{(k)}$ and $X_{(k+1)}$ and check that by a similar calculation.

Although we define the median for $n = 2k$ even as $(X_{(k)} + X_{(k+1)})/2$ to get a unique sample median, this problem shows that as far as minimizing sums of absolute values, the midpoint is no better than any other point in the interval between $X_{(k)}$ and $X_{(k+1)}$.

About PS4, problem 4: the introduction 'In the handout...it is stated that' seems to be of no help, since I can't find the statement there either. (Maybe it refers to an earlier, out of print edition of the handout?) So just ignore that.

In problem 4, you don't need to do scale-adjustment.

To say that Y_i's are symmetrically distributed around some M means that they all are, not just the ones different from X's. And it doesn't refer to a probability distribution they might have come from but to the sample itself. It means that for each Y_i (not equal to M) there is another one symmetrically on the oppposite side of M, in other words for some j, $Y_i - M = M - Y_j$, or $Y_j = 2M - Y_i$. It also means that if there are ties, so some Y_i are equal to each other, then there are just as many Y_j at $2M - Y_i$ as there are at Y_i. Hint: one may have the unchanged X_i all less than M and "new" Y_i larger than or equal to M.

If you have such a sample, then M must be its sample median, as you should show. Hint: if the sample size is odd, there must be at least one, and an odd number of observations equal to M.

The second part of the problem is to show that M is also the M-estimator of location for the Y_i sample, in other words $(\theta^* + \theta^{**})/2$, for any ψ function as in the Theorem in the handout.

Samples symmetrically distributed about a point might seem quite strange and unlikely to arise in real data. That's true, but the problem is showing what bad things can happen if you're allowed to change arbitrarily half or more of the data and put them wherever you want, as in the definition of breakdown point. A center of symmetry like M will be the location estimator for any affinely equivariant location estimator, actually, not only those defined by ψ functions.

In problem 5(b), to clarify, "M-estimator" should be "scale-adjusted M-estimator of location," as defined in the handout.