

18.443. Homework 4. Due Monday, October 6th.

(1) (5.10, No. 9) A manufacturer believes that defective products are produced with unknown probability p , which will be modeled as having a beta distribution. The manufacturer thinks that p should be around 0.05, but if the first 10 observed products were all defective, the mean of p would rise to 0.9. Find the beta distribution that has these properties.

(2) (6.2, No. 2) Suppose the proportion p of defective items in a large manufactured lot is known to be either 0.1 or 0.2, and the prior p.f. of p is as follows:

$$\xi(0.1) = 0.7 \text{ and } \xi(0.2) = 0.3.$$

Suppose also that when eight items are selected at random from the lot, it is found that exactly two of them are defective. Determine the posterior p.f. of p .

(3) (6.2, No. 4) Suppose that the prior of some parameter θ is a gamma distribution for which the mean is 10 and the variance is 5. Determine the prior p.d.f. of θ .

(4) (6.3, No. 12) Suppose that the time in minutes required to serve a customer at a certain facility has exponential distribution $E(\alpha)$ with α unknown and the prior of α is a gamma distribution with mean 0.2 and the standard deviation 1. If the average time required to serve a sample of 20 customers is 3.8 minutes, what is the posterior distribution of α .

(5) (6.4, No. 2) Suppose that a proportion p of defective items in a large shipment is unknown, and the prior of p is beta distribution with the parameters $\alpha = 5, \beta = 10$. Suppose that 20 items are selected at random from the shipment, and that exactly one of these items is found to be defective. If the squared error loss function is used, what is the Bayes estimate of p ?

(6) (6.4, No. 5) Suppose that the number of defects in a roll of magnetic recording tape has Poisson distribution $\Pi(\lambda)$ with λ unknown, and the prior of λ is a gamma distribution with parameters $\alpha = 3, \beta = 1$. On five randomly selected rolls the numbers of defects were to be 2, 2, 6, 0, 3. If the squared loss function is used what is the Bayes estimate of λ ?