18.440 PROBLEM SET 8: DUE APRIL 25

A. FROM TEXTBOOK CHAPTER SEVEN:

1. Problem 51: The joint density of X and Y is given by $f(x,y) = \frac{e^{-y}}{y}$, $0 < x < y$, $0 < y < \infty$. Compute $E[X^3|Y = y]$.
2. Problem 67: Consider a gambler who, at each gamble, either wins or loses her bet with respective probabilities p and $1-p$. A popular gambling system knkown as the Kelly strategy is to always bet the fraction $2p-1$ of your current fortune when $p > 1/2$. Compute the expected fortune after n gambles of a gambler who starts with x units and employs the Kelly strategy.
3. Problem 76: Let X be the value of the first die and Y the sum of the values when two standard (six-sided) dice are rolled. Compute the joint moment generating function of X and Y.
4. Theoretical Exercise 29: Let $X_1, \ldots, X_n$ be independent and identically distriuted random variables. Find
$$E[X_1|X_1 + \ldots + X_n = x].$$
5. Theoretical Exercise 36: One ball at a time is randomly selected from an urn containing a white and b black balls until all of the remaining balls are of the same color. Let $M_{a,b}$ denote the expected number of balls left in the urn when the experiment ends. Compute a recursive formula for $M_{a,b}$ and solve when $a = 3$ and $b = 5$.
6. Theoretical Exercise 48: If $Y = aX + b$, where a and b are constants, express the moment generating function of Y in terms of the moment generating function of X.
7. Theoretical Exercise 52: Show how to compute $\mathrm{Cov}(X,Y)$ from the joint moment generating function of X and Y.
8. Theoretical Exercise 54: If Z is a standard normal random variable, what is $\mathrm{Cov}(Z, Z^2)$?

B. (Just for fun — not to hand in) Let $V = (V_1, V_2, \ldots, V_n)$ be a random vector whose components V_i are independent, identically distributed normal random variables of mean zero, variance one. Note that the density function for V may be written as $f(v) = (2\pi)^{-n/2}e^{-|v|^2/2}$ where $v = (v_1, v_2, \ldots, v_n)$ and $|v|^2 = v_1^2 + v_2^2 + \ldots + v_n^2$.

1. Let M be an $n \times n$ matrix. Write $W = MV$ and compute the mean and covariance of W_i for each $1 \leq i \leq n$.

2. Write the probability density function for W.

3. Classify the set of matrices M for which MV has the same probability density function as V.

4. Is every n-dimensional mean zero multivariate normal distribution (as defined in Section 7.8 of the textbook) the distribution of MV for some choice of M? If so, to what extent does the distribution uniquely determine M?

C.(Just for fun — not to hand in) Try to formulate and prove a version of the central limit theorem that shows that sums of independent heavy-tailed random variables (divided by appropriate constants) converge in law to a stable random variable (instead of a normal random variable). See for example `http://eom.springer.de/A/a013920.htm` or the wikipedia articles on stable distributions for definitions and hints. You will need to use characteristic functions instead of the moment generating function.

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