MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Quantum Computation

Problem 1. Single-qubit σ_X errors can project the codeword $\alpha|000\rangle + \beta|111\rangle$ onto one of the following subspaces: $\{|000\rangle,|111\rangle\}$, $\{|100\rangle,|011\rangle\}$, $\{|010\rangle,|101\rangle\}$, and $\{|001\rangle,|110\rangle\}$. Construct a quantum circuit that specifies in which subspace the received codeword is. You can use two work qubits, some operations on the work qubits and the original qubits, and finally, a measurement on the work space.

Problem 2. Show how to correct a single σ_Z error for the phase-error correcting code:

$$|0\rangle \rightarrow \frac{1}{2}(|000\rangle + |110\rangle + |101\rangle + |011\rangle)$$

$$|1\rangle \rightarrow \frac{1}{2}(|111\rangle + |001\rangle + |010\rangle + |100\rangle).$$

Problem 3. For the Shor's nine-qubit code:

$$\begin{aligned}|0\rangle \rightarrow \frac{1}{2}(&|000\rangle|000\rangle|000\rangle + |000\rangle|111\rangle|111\rangle \\ &+|111\rangle|000\rangle|111\rangle + |111\rangle|111\rangle|000\rangle) \\ |1\rangle \rightarrow \frac{1}{2}(&|000\rangle|000\rangle|111\rangle + |000\rangle|111\rangle|000\rangle \\ &+|111\rangle|000\rangle|000\rangle + |111\rangle|111\rangle|111\rangle),\end{aligned}$$

give a quantum circuit that corrects a possible single-qubit Pauli error.

Problem 4. For the quantum Hamming code, show that the vector $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$ gets encoded to

$$\frac{1}{4}\left(\sum_{x\in\{H\}} |x\rangle - \sum_{x\in\{G\}-\{H\}} |x\rangle\right),$$

where $\{H\}$ is the corresponding binary subspace spanned by

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

and $\{G\} - \{H\}$ is the set of complements of $\{H\}$, which is all elements of $\{H\}$+[1 1 1 1 1 1 1].