2-D NEWTON

To follow up on that $x = x^2 + y^2$, $y = x^2 - y^2$ problem, let

$$f(x,y) = x^2 + y^2 - x \quad \text{and} \quad g(x,y) = x^2 - y^2 - y \ .$$

Then the partial derivatives are

$$f_x = 2x-1 \ , \quad f_y = 2y \ , \quad g_x = 2x \ , \quad g_y = -2y-1 \ ,$$

and in the tangent-plane spirit akin to 1-D Newton we seek increments Δx, Δy such that

$$f + f_x \Delta x + f_y \Delta y = 0 \quad \text{and} \quad g + g_x \Delta x + g_y \Delta y = 0 \ ,$$

to iterate our guesses via $x_{n+1} = x_n + \Delta x$, $y_{n+1} = y_n + \Delta y$.

Starting from $x_0 = 1$, $y_0 = 0.5$, the Fortran program **TRIFLE** from overleaf thus yields the following happy convergence:

n	x_n	y_n
0	1.000000000000	0.500000000000
1	0.812500000000	0.**4**37500000000
2	0.**77**3719879518	0.**4**20557228916
3	0.**771848**952637	0.**419645**658001
4	0.**7718445063**71	0.**4196433776**20
5	0.**771844506346**	0.**419643377607**
6	0.**771844506346**	0.**419643377607**

And even from $x_0 = 3$, $y_0 = 2$ the convergence is not too ghastly:

n	x_n	y_n
0	3.000000000000	2.000000000000
1	1.734693877551	1.081632653061
2	1.122860195064	0.650088930202
3	0.857703421403	0.**4**72990667534
4	0.**77**9859591015	0.**4**24383048522
5	0.**771**927503042	0.**4196**90800825
6	0.**771844**515361	0.**4196433**82652
7	0.**771844506346**	0.**419643377607**
8	0.**771844506346**	0.**419643377607**

Program **TRIFLE**

```
          implicit double precision (a-h,o-z)

          x = 1
          y = 0.5

          write (*,12) x,y
12        format (//' Need  x,y =', 2f10.4)

          read (*,*) x,y

          do 29 n=0,10

            write (*,25) n, x,y
25          format (i10, 2f18.12)

            f = x*x + y*y - x
            g = x*x - y*y - y

            fx =  2*x - 1
            fy =  2*y

            gx =  2*x
            gy = -2*y - 1

            denom = fx * gy - fy * gx

            dx = (g * fy - f * gy) / denom
            dy = (f * gx - g * fx) / denom

            x = x + dx
            y = y + dy

29        continue

          end
```