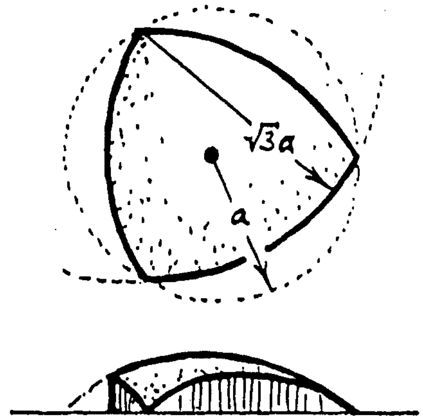
18.330 Problem Set 6 — Firmly due in class: Mon 29 Mar 04

(16) Calculate the **roof area** of Kresge auditorium, working from the theory that this familiar object is shaped like a paraboloid of revolution

$$z = (a^2 - x^2 - y^2) / 2a ,$$

truncated by vertical cylinders of radius $\sqrt{3}\,a$ centered on the opposite vertices.



(17) In the spirit of **Gaussian quadrature**:

(a) determine **polynomials** $p_0(x)$, $p_1(x)$ and $p_2(x)$ such that

$$\int_0^1 \sqrt{x}\; p_m(x)\; p_n(x)\; dx = 0 \quad \text{for} \quad m \neq n ,$$

(b) find **weights** w_1 and w_2 such that the estimate

$$\int_0^1 \sqrt{x}\; f(x)\; dx = w_1\, f(x_1) + w_2\, f(x_2)$$

based on the roots x_1 and x_2 of $p_2(x)$ becomes exact for all **cubic** polynomials, and

(c) finally **test** this fancy folderol on the integral $\int_0^1 \sqrt{\sin x}\; dx$.

(18) Evaluate the sum $$S = \sum_{k=1}^{\infty} (1/x_k)^2 ,$$

where x_k is the k-th positive root of $x = \tan x$.

Work carefully here, and employ sensible extrapolations or some other finesse like

$$1 + 1/9 + 1/25 + 1/49 + 1/81 + \ldots = \pi^2/8 .$$

Then you should find that this sum S equals a very simple fraction!