Pak

9/14/05

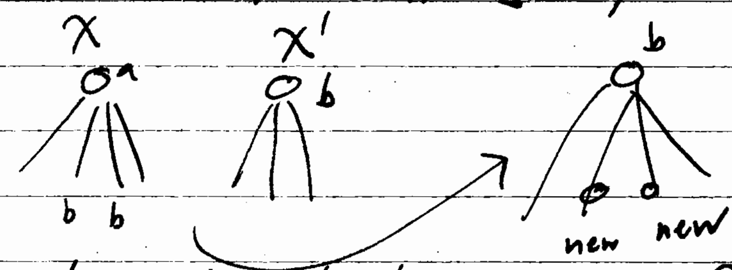
Today: Combinatorics Seminar (see webpage)

More on colorings

Thm G graph on n vertices, max deg $= d$
Let χ, χ' be proper K-colorings.
One can obtain χ from χ' by changing one color at a time. (For $K \geq d+2$)

Pf of thm: Kinda like greedy alg

pick



note changed children can't be farther from χ' since o/w they're b in χ' ↯ ✓

Thm χ, χ' proper 3-colorings of G_{mn}, same thing

Pf: First off, we restrict ourselves to colorings w/ fixed 0-1 boundary (it's true in general, but we'll restrict)

Construct height function $f: G_{mn} \to \mathbb{Z}$ and $f(x) = f(y) \pm 1$ $(x,y) \in G_{mn}$ $f/_{3\mathbb{Z}} = \chi$

Claim: $\exists! f$ (up to addition of $3\mathbb{Z}$) (and assuming $f(1,1) = 0$)

Pf: Do greedy construction. At each stage, must be ~~adjacent~~ same mod 3 + in same class as prev

Lemma Every coloring is conn. to a (the) 0-1 coloring

Pf: replace local max

$$\begin{array}{ccc} & x & \\ x & x+1 & x \\ & x & \end{array} \rightarrow \begin{array}{ccc} & x & \\ x & x-1 & x \\ & x & \end{array} \quad ✓$$

Thm (Erdős) (1963) (also ∈ Jukna)

$\mathcal{F}$ family w/ all sets $\geq K$, $|\mathcal{F}| < 2^{K-1}$.

Then $\mathcal{F}$ is 2-colorable

Pf: Consider all 2-colorings of $[n]$.

χ is bad if ~~at least~~ $\exists\, S \in \mathcal{F}$ monoch.

Note # bad colorings $< 2^n \Rightarrow$ ✓. Now, for each $S \in \mathcal{F}$ $\exists \leq 2^{n-K+1}$, ta-da! ✓

State + talk about, but don't prove, next thm:

Thm (Erdős - Lovász 1974)

Let $\mathcal{F}$ be K-uniform + intersecting.

$|\mathcal{F}| > K^K \Rightarrow \mathcal{F}$ is 2-colorable

Def'n: K-uniform: $S \in \mathcal{F} \Rightarrow |S| = K$

intersecting $S_1, S_2 \in \mathcal{F} \Rightarrow S_1 \cap S_2 \neq \emptyset$