

Recap extended theory:

1. smooth parts: use pde and characteristics.
2. when characteristics cross: introduce shocks. shock speed = $[q]/[\rho]$
3. IMPORTANT: use shocks only to avoid characteristic crossings.
Characteristics must end at the shock. Mathematical formulation of this:
ENTROPY CONDITION.

Explain how this:

4. Allows construction of solution for all times. But need to identify where characteristics first cross, to insert a shock there.
5. Introduces the arrow of time: solutions with shocks are NOT TIME REVERSIBLE.

Example of a place where NO SHOCK is needed: rarefaction.

NOT ALL DISCONTINUITIES THAT SATISFY speed = $[Q]/[\rho]$ ARE ALLOWED!

Example 2: $A_t + (0.5*A^2)_x = 0$ [Flood waves with $Q = 0.5*A^2$] with IC:

$A = 0$ for $x > 1$, and $A = A_0$ for $x < 0$.

A linear between $x=0$ and $x=1$.

Show condition is $s = \text{average}(A)$.

Compute characteristics. Draw, and find crossing. Introduce shock.

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