18.310C Pre-recitation Assignment 1

Reading: Sections 1, 2 and 5 of the lecture notes on Probability. You will need these readings to complete the assignment.

Instructions: Hand in written solutions to the following questions **at the begininning of recitation.**

1. A box contains 7 black balls and 4 white balls. Two balls are randomly drawn from the box. Let X_1 be the random variable denoting the color of the first ball and X_2 denote the color of the second ball. Define an appropriate sample space for this random process. Using your definition, compute the probabilities of the following events by describing the subsets of the sample space to which they correspond:

   - $(X_1, X_2) = (\text{WHITE}, \text{BLACK})$.
   - $\{X_1, X_2\} = \{\text{WHITE}, \text{BLACK}\}$.

   A note on notation: The notation $(a_1, \ldots, a_n)$ indicates an *ordered* n-tuple, while the notation $\{a_1, \ldots, a_n\}$ indicates a set of n elements. In particular, notice that $(a, b) \neq (b, a)$ and $\{a, b\} = \{b, a\}$.

2. Consider now the same setting. Initially the box contains again 7 black balls and 4 white balls. Suppose that we repeatedly draw a ball at random from the box, observe its color and then discard it. We do this 4 times. For $i \in \{1, 2, 3, 4\}$, let X_i be the random variable representing the color of the ith ball drawn from the box.

   Question: Is it true that

   $$\mathbb{P}\left(\{X_1, X_2\} = \{\text{WHITE}, \text{BLACK}\}\right) = \mathbb{P}\left(\{X_3, X_4\} = \{\text{WHITE}, \text{BLACK}\}\right)?$$

   Write your best guess for the answer to this question and a few short sentences explaining how you came to this conclusion.

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