18.307: Integral Equations | **M.I.T. Department of Mathematics**
Spring 2006

Homework 2 | **Due:** Wednesday, 03/08/06

4. (Read Sec. 2.4 in text by M. Masujima.) (a) Find a suitable Green's function for the time-dependent, one-dimensional Schrödinger equation

$$i\Psi_t(x,t) + \Psi_{xx}(x,t) = V(x,t)\,\Psi(x,t),$$

with the initial value $\Psi(x,0) = a(x)$.
(b) Express this initial-value problem in terms of an integral equation. In what case and how would you proceed to solve this equation approximately? Explain.

5. (Prob. 2.14 in text by M. Masujima.) (a) Find the Green's function for the partial differential equation

$$u_{tt} - u_{xx} = p - \lambda\, u_{xx}\, u_x^2, \quad u = u(x,t),\ -\infty < x < \infty,\ t > 0,$$

with the initial conditions $u(x,0) = a(x)$ and $u_t(x,0) = b(x)$; λ is a constant. This equation describes the displacement of a vibrating string under the distributed load $p = p(x,t)$.
(b) Express this initial-value problem in terms of an integral equation. Explain how you would find an approximate solution if λ were small.
Hint: Find a function $u_0(x,t)$ that satisfies the wave equation, i.e., $u_{0,tt} - u_{0,xx} = 0$, and the given initial conditions.

6. (Prob. 3.4 in text by M. Masujima.) Consider the Volterra equation of the first kind

$$f(x) = \int_0^x dy\, K(x-y)\, u(y), \quad f(0) = 0.$$

Solve the equation for $K(x) = \ln x$ and arbitrary (admissible) $f(x)$. **Hint:** $\int_0^\infty dx\, e^{-x} \ln x = -\gamma$, where $\gamma = 0.5772156649\ldots$ is Euler's constant.