18.306 Advanced Partial Differential Equations with Applications
Fall 2009

Lecture 05 2009 09 23 WED

TOPICS: Domains of influence and dependence.
Causality and uniqueness. Allowed boundary conditions.
Examples.

Domain of definition and domain of dependence: where is the solution defined.
Implications for where conditions must be given:
u_t + c(x)*u_x = 0 in an interval a < x < b.
Causality:
If c(a) > 0, BC's needed at x = a, and only then.
If c(b) < 0, BC's needed at x = b, and only then.
Draw characteristics for various example c = c(x).
Generalize method of characteristics to other first order scalar eqn.:
--- Semilinear.
--- Quasilinear.

Domain of definition of solution does not depend on data for linear.
Semilinear
Do example: x*u_x + y*u_y = u^2, with u(x, 1) = F(x)
Domain of definition depends on F [solution blows up along characteristics when F not zero].
Do example u_t + c*u_x = u^2, with u(x, 0) = F(x).
Solution not defined for all t > 0 along characteristics where F > 0.
Quasilinear
Characteristics may cross, leading to multiple values.
Start with u_t + c(u)*u_x = 0 and u(x, 0) = F(x).
Solutions by characteristics.
Implicit form of the solutions.
Crossing of characteristics.