Lecture 14

(See notes.) Introduced Green's functions by analogy with matrix inverses, and constructed Green's function of $-d^2/dx^2$ with Dirichlet boundaries as an example.

We had to jump through some hoops to avoid a problematic-looking "delta function" that keeps appearing, a limit of a function whose area is "infinitely concentrated" at a "single point". This is possible, but becomes more and more painful as we go on, motivating us to find an alternate definition of "function" in the future, a **distribution**.

For the 1d example, we can explicitly check that $u(x)=\int G(x,x')f(x')dx'$ solves $-u''=f$. (See 2nd handout.)

Further reading: Strang book, section 1.4. Many PDE books introduce Green's functions and delta functions in various ways; see, e.g. section 9.3.4 of *Elementary Applied Partial Differential Equations* by Haberman.

18.303 Linear Partial Differential Equations: Analysis and Numerics

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