18.112 Functions of a Complex Variable
Fall 2008

# Lecture 4: Power Series

(Text 33-42)

## Remarks on Lecture 4

## Problem 8 on p.41

We know $\sum_0^\infty w^n$ converges only for $|w| < 1$. Otherwise the terms do not converge to 0. Now put

$$z' = z + \frac{1}{2},$$

so

$$w = \frac{z}{1+z} = \frac{z' - \frac{1}{2}}{z' + \frac{1}{2}}.$$

So $|w| < 1$ is equivalent to

$$\text{Re} z' > 0,$$

or equivalently

$$\text{Re} z > -\frac{1}{2}.$$

## Problem 9 on p.41

Write

$$\frac{z^n}{1+z^{2n}} = \frac{1}{z^n + z^{-n}}.$$

Write $a_n \sim b_n$ if

$$\left|\frac{a_n}{b_n}\right| \longrightarrow c \neq 0.$$

Then if $|z| > 1$,

$$\frac{1}{z^n + z^{-n}} \sim z^{-n},$$

and if $|z| < 1$,

$$\frac{1}{z^n + z^{-n}} \sim z^n.$$

So in both cases we have convergence. If $z = e^{it}$, we have

$$\frac{1}{z^n + z^{-n}} = \frac{1}{2\cos nt},$$

so the terms do not tend to 0, so we have divergence.