18.100C Lecture 22 Summary

Lemma 22.1. *If f is Riemann-Stjeltjes integrable on $[a, b]$, then the same holds for any smaller closed interval. Moreover, for any $c \in (a, b)$,*

$$\int_a^c f\, d\alpha + \int_c^b f\, d\alpha = \int_a^b f\, d\alpha.$$

Here are two different versions of the Fundamental Theorem of Calculus:

Theorem 22.2. *Suppose that $f : [a, b] \to \mathbb{R}$ is Riemann integrable, and define*

$$F(x) = \int_a^t f(t)\, dt.$$

Then F is continuous. Moreover, if f is continuous at some point x_0, then F is differentiable there, and $F'(x_0) = f(x_0)$.

Theorem 22.3. *Suppose that $f : [a, b] \to \mathbb{R}$ is differentiable, and f' is Riemann integrable. Then*

$$\int_a^b f'(x)\, dx = f(b) - f(a).$$

Reminder: radius of convergence of a power series.

Lemma 22.4. *Suppose that $f(x) = \sum_{k=0}^{\infty} a_k x^k$ has radius of convergence $\rho > 0$. Then f is differentiable at all points $x \in (-\rho, \rho)$, and its derivative is*

$$f'(x) = \sum_{k=1}^{\infty} a_k k\, x^{k-1},$$

which is a power series with the same radius of convergence ρ.

Example 22.5. *The function $f(x) = x + x^2/2 + x^3/x + \cdots$, with radius of convergence 1, satisfies $f'(x) = 1/(1 - x)$.*