18.085: Matlab Homework #4

Differential Equations.

- Find the exact solution to

$$-\frac{d}{dx}\left(c\frac{du}{dx}\right) = f(x)$$

  where $f(x)$ is a delta function at $x = 1/2$ and $c = 1$. The boundary conditions are $u(0) = 0$ and $w(1) = 0$ $(0 \leq x \leq 1)$.

- Now investigate the solution numerically using MATLAB. Solve

$$A^T C A u = f$$

  where A is the discrete approximation to the derivative operator. Look at *TWO* cases: (1) boundary conditions the same as above; (2) $u(0) = u(1) = 0$. Try these two cases where A comes from a "centered difference" scheme:

$$\left(\frac{du}{dx}\right)_i \approx \frac{u_{i+1} - u_{i-1}}{2\Delta x}$$

  and with an "upwinded" scheme:

$$\left(\frac{du}{dx}\right)_i \approx \frac{u_i - u_{i-1}}{\Delta x}.$$

  Comment on the results in all four cases.

- Now add a perturbation to the original equation:

$$-\frac{d}{dx}\left(c\frac{du}{dx}\right) + \epsilon u = f(x)$$

  where ϵ is very small. Again, solve this numerically and compare with the solutions above. Does ϵ appear to be a regular or a singular perturbation?

- Plot all of your results and compare them to the exact solution found above. (Try this for a few different values of N where N is the number of grid points.)