Justify your answers. Cross out what is not meant to be part of your final answer. Total number of points: 50

I. (5 pts) Find all solutions of the equation

$$z^{4/3} = 1 + i.$$

II. (Total 10 pts)

1. (3 pts) Can the function $v(x, y) = 4xy + x + y$ be the imaginary part of an analytic function?

2. (5 pts) Determine all the functions $u(x, y)$ such that $u(x, y) + iv(x, y)$ is analytic.

3. (2 pts) Find $f(z)$ such that

$$f(z) = u(x, y) + iv(x, y).$$

III. (5 pts) Compute the line integral

$$\int_C \frac{(z^2 - 2)}{z^3} dz$$

where C is the left half–circle joining $-i$ and i.

IV. (Total 15 pts) Let

$$f(z) = \frac{z}{(1 - z)(z + 3)}.$$

1. (3 pts) Write $f(z)$ as a sum of fractions, i.e.,

$$f(z) = \frac{A}{z - 1} + \frac{B}{z + 3};$$

2. (2 pts) Find the singularities of $f(z)$ and classify them;

3. (3 pts) Explain whether it is possible to expand f(z) in Laurent (or Taylor) power series of:

   (i) $z + 2$, that converges in the region $1 < |z + 2| < 4$?

   (ii) $z + 2$, that converges in the region $3 < |z + 2|$?

   (iii) $z + 2$, that converges in the region $1 < |z + 2| < 3$?

4. (7 pts) Write the Laurent series expansion of $f(z)$ in $|z - 1| < 2$ as a power series of $(z - 1)$.

V. (Total 8 pts)

1. (5 pts) Let
$$f(z) \;=\; \frac{1}{(z^2 - z - 2)^2}.$$

Find the singularities of $f(z)$, classify them, and compute the residues of $f(z)$ at those singular points.

2. (3 pts) Compute the integral of $f(z)$ along the circles of center 0 and radii $1/4$, $5/4$, and 4, respectively.

VI. (Total 7 pts)

1. (2 pts) Show that the function $f(z) = \cos z^{1/2}$ is analytic at $z = 0$.

2. (5 pts) Show that the function
$$f(z) = \frac{\cos z - 1}{\sinh z - z}$$

has a simple pole at the origin.