

# Comparison of Bayesian and Frequentist Inference

18.05 Spring 2014

- First discuss last class 19 board question,

# Compare

## **Bayesian inference**

- Uses priors
- Logically impeccable
- Probabilities can be interpreted
- Prior is subjective

## **Frequentist inference**

- No prior
- Objective –everyone gets the same answer
- Logically complex
- Conditional probability of error is often misinterpreted as total probability of error
- Requires complete description of experimental protocol and data analysis protocol before starting the experiment. (This is both good and bad)

## Concept question

Three different tests are run all with significance level  $\alpha = 0.05$ .

1. Experiment 1: finds  $p = 0.03$  and rejects its null hypothesis  $H_0$ .
2. Experiment 2: finds  $p = 0.049$  and rejects its null hypothesis.
3. Experiment 3: finds  $p = 0.15$  and fails to reject its null hypothesis.

Which result has the highest probability of being correct?

(Click **4** if you don't know.)

**answer: 4.** You can't know probabilities of hypotheses based just on  $p$  values.

## Board question: Stop!

Experiments are run to test a coin that is suspected of being biased towards heads. The significance level is set to  $\alpha = 0.1$

**Experiment 1:** Toss a coin 5 times. Report the sequence of tosses.

**Experiment 2:** Toss a coin until the first tails. Report the sequence of tosses.

1. Give the test statistic, null distribution and rejection region for each experiment. List all sequences of tosses that produce a test statistic in the rejection region for each experiment.

2. Suppose the data is *HHHHT*.

(a) Do the significance test for both types of experiment.

(b) Do a Bayesian update starting from a flat prior:  $\text{Beta}(1,1)$ .

Draw some conclusions about the fairness of coin from your posterior.

(Use R: `pbeta` for computation in part (b).)

## Board question: Stop II

For each of the following experiments (all done with  $\alpha = 0.05$ )

- (a) Comment on the validity of the claims.  
(b) Find the true probability of a type I error in each experimental setup.

- 1 By design Ruthi did 50 trials and computed  $p = 0.04$ .  
She reports  $p = 0.04$  with  $n = 50$  and declares it significant.
- 2 Ani did 50 trials and computed  $p = 0.06$ .  
Since this was not significant, she then did 50 more trials and computed  $p = 0.04$  based on all 100 trials.  
She reports  $p = 0.04$  with  $n = 100$  and declares it significant.
- 3 Efrat did 50 trials and computed  $p = 0.06$ .  
Since this was not significant, she started over and computed  $p = 0.04$  based on the next 50 trials.  
She reports  $p = 0.04$  with  $n = 50$  and declares it statistically significant.

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## 18.05 Introduction to Probability and Statistics

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