18.04 Recitation 1
Vishesh Jain

1.1. Is is true that $e^{\log(z)} = z$? Is is true that $\log(e^z) = z$?

Ans: The first part is true – compute directly. The second part is on problem set 2.

1.2. If you know one value of $\log(z)$, what are all the other values?

Ans: The other values differ by $i2n\pi$.

2. Let $z_1 = 2e^{i\pi/3}$ and $z_2 = 3 + 4i$.

2.1. Compute $\log(z_1)$. What is the value in the principal branch?

Ans: $\log(2) + i\pi/3 + i2n\pi$. The value in the principal branch is $\log(2) + i\pi/3$.

2.2. Compute $\log(z_2)$. What is the value in the principal branch?

Ans: $\log 5 + i\theta + i2n\pi$ where $\theta = \arctan(4/3)$.

3.1. Is z^a single valued or multi-valued? Why?

Ans: Multivalued; this comes from the multivalued nature of log.

3.2. Suppose $z \neq 0$. Is z^a single valued or multi-valued when a is an integer?

Ans: Single-valued. Check directly using the definition $z^a = e^{a \log z}$.

3.3. If a is a real number, what do all the a^{th} powers of z have in common?

Ans: They have the same absolute value.

3.4. If a is a purely imaginary number, what do all the a^{th} powers of z have in common?

Ans: They have the same principal value for the argument.

4. Let $z_1 = 2e^{i\pi/3}$ and $z_2 = 3 + 4i$.

4.1. Compute $z_1^{z_2}$.

Ans: $e^{3\log 2 - 4(\pi/3)} e^{i(4\log 2 + \pi)} e^{-8n\pi} e^{i6n\pi}$

4.2. Compute $z_1^{1/4}$. How many distinct values do you get? Plot all these values in the complex plane.

Ans: We get the following four distinct values: $e^{\{\log(2) + i\pi/3 + i2n\pi\}\frac{1}{4}} = 2^{1/4} e^{i\pi/12}\{1, e^{i\pi/2}, e^{i\pi}, e^{i3\pi/2}\}$.

18.04 Complex Variables with Applications

Spring 2018