18.04 Practice problems for final exam, Spring 2018

On the final exam you will be given a copy of the Laplace table posted with these problems.

Problem 1.
Which of the following are meromporphic in the whole plane.

(a) z^5

(b) $z^{5/2}$

(c) $e^{1/z}$

(d) $1/\sin(z)$.

Problem 2.
(a) Let $f(z) = \dfrac{(z-2)^2 z^3}{(z+5)^3(z+1)^3(z-1)^4}$. Compute $\displaystyle\int_{|z|=3} \frac{f'(z)}{f(z)}\,dz$

(b) Find the number of roots of $g(z) = 6z^4 + z^3 - 2z^2 + z - 1 = 0$ in the unit disk.

(c) Suppose $f(z)$ is analytic on and inside the unit circle. Suppose also that $|f(z)| < 1$ for $|z| = 1$. Show that $f(z)$ has exactly one fixed point $f(z_0) = z_0$ inside the unit circle.

(d) True or false: Suppose $f(z)$ is analytic on and inside a simple closed curve γ. If f has n zeros inside γ then $f'(z)$ has $n-1$ zeros inside γ.

Problem 3.
Let $A = \{z|\, 0 \le \mathrm{Re}(z) \le \pi/2,\ \mathrm{Im}(z) \ge 0$.

Let $B =$ the first quadrant/

Show that $f(z) = \sin(z)$ maps A conformally onto B

18.04 Complex Variables with Applications

Spring 2018