PROFESSOR: So in this recitation, we're going to look at step and delta functions, integration and generalized derivatives. So the first part, you're asked to compute the integral from zero minus to infinity delta(t) exponential t squared dt. The second one is from zero minus to infinity delta(t-2) exponential of t squared sine t cos 2t. The third one is zero plus to infinity delta(t) exponential of squared dt. So know that it's the same as a, except that the bounds of integration changed.

The second part, we're asked to define the generalized derivatives of these two functions, where u here is just a step function that you saw before. So it's 3u(t) minus 2u(t-1). And the second one is t squared for t is negative and exponential of minus t for t positive. So why don't you pause the video and work through this example, and I'll be right back.

Welcome back. So let's compute the first one. Zero minus, infinity, delta(t) exponential t squared dt. So just to remind you, the delta function is everywhere zero except at the value zero. And we represent it with an arrow. And the integral of the delta would be 1 from minus infinity to plus infinity.

In this integral, we're integrating from zero minus to infinity, which means that the zero is included in our interval from zero minus to infinity. Therefore, this integral is basically assigning the value to this function exponential t squared. And the value that it's assigning to it is the value it would take at t equals to zero where the delta is non-zero. So really clearly, this is just exponential of zero. And it gives us 1.

For the second integral, it goes from zero minus to infinity, delta t minus 2 exponential, a more complicated function, t square sine t cos 2t. So now, let's represent this delta function here. So this is just our zero axis. And the delta here is zero everywhere except at 2 where we would represent it, again, with an arrow at 2 amplitude 1.

So this delta is zero everywhere except at 2 where it would assign the value to the function next to it at the value t equals to 2. So really this integration gives us just the value of this function at t equal 2, so 4 sine 2 cos 4. And here, the key was that again, this interval of integration from zero minus to infinity and clearly it includes the value at which delta function is non-zero.

So for the last one, we return to our first integral except that now we are changing the bounds of integration to zero plus to infinity. So now, if I do representation of the delta function that we're dealing with, so delta centered at 1, and the interval of integration, we have an open interval now that does not include the value at which delta is non-equal to zero. So everywhere this function would just be assigned its value at-- it'd just be multiplied by the function that is zero. And so basically, it's like multiplying this function by zero, and it just gives us zero. It's like the delta fell off of our interval of integration, so we're just left with a zero function.

So let's move to the second part. The second part asked us to find a generalized derivative to f of t equals 3u of t minus 2u of t minus 1. So just to remind you here of what u of t's are, just want to sketch a few things. So first, u of t is just the step function that would be zero everywhere and would take the value 1 for t larger than zero. So this first part here would just be non-zero for t larger than zero. Instead of being assigned value 1, it's just the assigned value 3 because we're multiplying the u of t by 3.

So this first part would look like this. The second part here would be u shifted by minus 1, which means that u is zero everywhere for t less than 1. So we would have a zero function here. So let me just do dots, but it should be on the same axis. And 1 for t larger than 1. But here, we're multiplying it by factor minus 2. So really, what we have is another u function that is shifted down to minus 2.

So the sum of these two contributions is zero for t negative, takes the value 3 for t between zero and 1, and the value 1, 3 minus 2, for t larger than 1. So clearly here, we have discontinuities at t equals to zero and t equals to 1. So let's just write down the derivative.

The generalized derivative here would lead us first to compute the derivatives where the function is continuous. So for minus infinity to zero, it's a constant, derivative would be zero. Between zero and 1, it's constant, derivative would be zero. And from 1 to infinity, it would also give us zero. So we would have a zero contribution from the continuous part of the function, if you wish.

But we still need to account for the discontinuities. So at zero, we have a jump from zero to 3. And that we learned can be written down as a delta function of magnitude 3 centered at zero. After that, we have another jump. Now, it's from 3 to 1. So it's a jump of minus 2 amplitude centered at 1. So here, we can also do that with the delta, but we just need also to shift it by minus 1 to show clearly that the jump occurs at 1 and multiply this by minus 2 to show the amplitude of the jump down.

So if we were now to represent this f prime, basically, the regular part is zero. So there's nothing to write down except just a zero function. And these discontinuities that I'm just going to represent on the graph would be the delta function centered at zero magnitude 3 and delta function centered around 1 of magnitude minus 2. And the rest would just be the zero function. So that would be f prime. f of t.

So for the second one that we were given, it's a function f of t that takes the value t squared for t negative and exponential minus t for t positive. So a quick sketch here tells that this function looks like this, an exponential minus t for t positive taking a value here 1. So clearly, there is a jump here, discontinuity.

So how do we go about computing this generalized derivative? So let's look again at the continuous parts. So from minus infinity to zero, we're dealing with just the t squared. So the derivative is just 2t. For t between zero and infinity, we're just dealing with exponential minus t. So that's minus exponential minus t. But we need to account for the discontinuity the jump of amplitude, 1 at zero.

So as we saw before, this can just be modeled with a delta function. And if we were to represent this function, then we would just need to add the delta function of magnitude 1 here and then just sketch 2t, for example, and then minus exponential of minus 2t. That would give us something like that.

And that ends the problem for today. And the key point here were just to learn how to manipulate the step function and how to use the delta function when you compute your integrals. And be careful with the bounds of integration.