18.034, Honors Differential Equations
Prof. Jason Starr
Lecture 27
4/12/04

1. Some amount of back-tracking and defining things rigorously. Defined $\mathbb{R}$ - and $\mathbb{C}$ - vector spaces V, Linear transformations T: $V \to W$, Ordered basis. Gave main examples: $V = \mathbb{R}^n$, A an mxn matrix, get T_A: $\mathbb{R}^n \to \mathbb{R}^m$ by x $\mapsto$ Ax, standard basis (e_1,…, e_n).

Given a linear transformation T: $V \to W$ and ordered basis B_V for V and B_W for V, introduced the notation $A = [T]_{B_W, B_V}$, $Tv_{ij} = \sum_i A_{ij} w_i$. Proved $[T]_{s+\partial, s+\partial} = A$ and $[T \circ S]_{B_W, B_V} = [T]_{B_W, B_V} \circ [S]_{B_V, B_U}$.

Talked about a change-of-basis matrix $[I\partial]_{B,B'}$ and proved that, given a basis B, a set B′ is a basis iff $[I\partial]_{B,B'}$ is invertible, in which case $[I\partial]_{B',B} = ([I\partial]_{B,B'})^{-1}$.

2. Defined the char. poly of a linear operator T: $V \to V$, $pT(\lambda) = \det(\lambda I_n - [T]_{B,B})$ and proved this is independent of the choice of B.

3. Explained notation q(A) where A is a square matrix and q(λ) is a polynomial.

4. Stated the Cayley-Hamilton theorem $p_A(A) = 0$.

5. Defined generalized eigenspaces.

6. Stated (but not yet proved) that /$\mathbb{C}$ the Cayley-Hamilton theorem implies that the generalized eigenspaces give a direct sum decomposition.