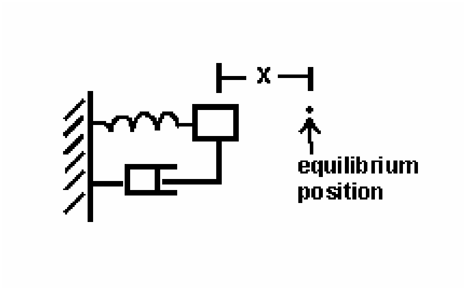
18.034, Honors Differential Equations
Prof. Jason Starr
Lecture 11
2/27/04

0. Handed back exams w/ brief discussion.

1. Introduced 2nd order ODE's w/ spring model



mx'' = Net force = Force of spring + Force of dashpot + Driving force $= -kx - bx' + F(t)$.

$$x'' + \frac{b}{m}x + \frac{k}{m} = F(t)$$

2. General 2nd order linear ODE in normal form

$y'' + a(t)y + b(t)y = f(t)$. <u>Homog</u>: if $f(t) = 0$
<u>Const. Coeff</u>: If a(t), b(t) are consts.

Discussed existence/ uniqueness/ maximal extension thm (follows from our earlier general thm + exponential bound on y(t)).

3. Homg. + constant coeff: $y'' + ay + by = 0$.

Motivated by 1st order case, guess $y(t) = C \bullet e^{rt}$. More generally, guess $y(t) = e^{rt} \bullet f(t)$ for some f(t).

Defined the characteristic poly P(r) assoc'd to a constant coeff., linear homog. ODE.

<u>Proved the E.S.R.</u> Let P(r) be a polynomial. Then $P\left(\frac{d}{dt}\right)e^{rt}f(t) = e^{rt}P\left(\frac{d}{dt} + r\right)f(t)$. So $y = e^{rt}f(t)$ is solution of $P\left(\frac{d}{dt}\right)y = 0$ iff f(t) is solution of $P\left(\frac{d}{dt} + r\right)$. In particular, if $P(r) = 0$, then Ce^{rt} is a solution.

Discussed 2nd order case. Proved if P(r) has 2 distinct roots r_1 & r_2, then $y(t) = C_1e^{r_1t} + C_2e^{r_2t}$ is a solution for every C_1 and C_2.
Proved, if P(r) has a repeated root r_1, then $y(t) = C_1e^{r_1t} + C_2e^{r_1t}$ is a solution for every C_1 & C_2.