18.03 Recitation 16, April 6, 2010

Step and delta functions, and step and delta responses

1. Let $Q(t) = \begin{cases} 0 & \text{for } t < 1 \\ 2t - 2 & \text{for } 1 < t < 2 \\ 2t - 1 & \text{for } 2 < t < 3 \\ 5 & \text{for } 3 < t \end{cases}$

(a) Sketch a graph of this function. Is it piecewise smooth?

(b) Find the generalized derivative $q(t) = Q'(t)$, and sketch it.

(c) Tell a story which might result in the equation $\dot{x} + kx = q(t)$ with rest initial conditions. (Your choice of k, it might be negative).

(d) Tell a story which might result in the equation $2\ddot{x} + 4\dot{x} + 4x = q(t)$ with rest initial conditions.

2. Find the unit step and unit impulse responses for the operator $2D + I$, and graph them.

3. Find the unit impulse response for the operator $D^2 + 2D$, and graph it.

4. From your answer to **3.**, find the solution to $\ddot{x} + 2\dot{x} = 3\delta(t - 1)$ with rest initial conditions.

18.03 Differential Equations
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