18.02 Practice Final 3hrs.

Problem 1. Given the points $P:(1,1,-1), Q:(1,2,0), R:(-2,2,2)$ find

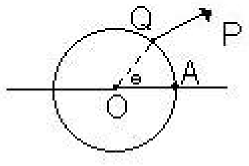
$a)PQ \times PR$ b) a plane $ax+by+cz=d$ trough P, Q and R

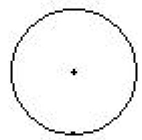
Problem 2. Let $\mathbf{A} = \begin{pmatrix} 1 & 0 & c \\ 2 & c & 1 \\ 1 & -1 & 2 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{A}^{-1} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \times & \cdot \end{pmatrix}$.

a) For what valu(s) of constant c will $\mathbf{Ax} = 0$ have a non-zero solution?

b) Take $c = 2$, and tell what entry the inverse matrix has in the position market $\times$

Problem 3. The roll of Scotch tape has outer radius a and is fixed in position (i.e., does not turn). Its end P is originally at the point A; the tape is then pulled from the roll so the free portion makes a 45-degree





angle with the horizontal.

Write the parametric equation $x = x(\theta) \quad y = y(\theta)$ for the curve C traced out by the point P as it moves. (Use vectore methods; θ is the angle shown)

Sketch the curve on the second picture, showing its behavior at its endpoints.

Problem 4. The position vectore of point P is $r =< 3\cos t, 5\sin t, 4\cos t >$.

a) Show its speed is constant.

b) At what point $A:(a,b,c)$ does P pass through the yz-plane?

Problem 5. Let $\omega = x^2y - xy^3$, and $P = (2,1)$

a)Find the directional derivative $\frac{d\omega}{ds}$ at P in the diraction of $\mathbf{A} = 3i + 4j$.

b) If you start at P and go a distance .01 in the diraction of $\mathbf{A}$, by approximately how much will ω change? (Give a decimal with one significant digit.)

Problem 6. a) Find the tangent plane at (1,1,1) to the surface $z^2 + 2y^2 + 2z^2 = 5$; give the equation in the form $az + by + cz = d$ and simplify the coefficients.

b) What dihedral angle does the tangent plane make with the xy-plane? (Hint: consider the normal vectors of the two planes.)

Problem 7. Find the point on the plane $2z + y - z = 6$ which is closest to the origin, by using Lagrange multipliers. (Minimize the square of the distance. Only 10 points if you use some other method)

Problem 8. Let $\omega = f(x,y,z)$ with the constraint $g(x,y,z) = 3$.

At the point $P:(0,0,0)$, we have $\nabla f =< 1,1,2 >$ and $\nabla g =< 2,-1,-1 >$, Find the value at P of the two quantities (show work): a) $\left(\dfrac{\partial z}{\partial x}\right)_y$ b) $\left(\dfrac{\partial \omega}{\partial x}\right)_y$

Problem 9. Evaluate by changing the order of integration: $\int_0^3 \int_{z^3}^9 xe^{-y^2}\,dydz$.

Problem 10. A plane region R is bounded by four semicircles of radius 1. having ends at $(1,1), (1,=1), (-1,1), (-1,-1)$ and centerpoints at $(2,0), (-2,0), (0,2), (0,-2)$.

Set up an iterated integral in polar coordinates for the moment of inertia of R about the origin; take the density $\delta = 1$. Supply integrand and limits, but *do not evaluate* the integral.

Use symmetry to simplify the limits of integration.

Problem 11. a) In the xy-plane, let $\mathbf{F} = P\mathbf{i} + Q\mathbf{j}$. Give in terms of P and Q the line integral representing the flux $\mathbf{F}$ across a simple closed curve C, with outward-pointing normal.

b) Let $\mathbf{F} = ax\mathbf{i} + by\mathbf{j}$. How should the constants a and b be related if the flux of $\mathbf{F}$ over any simple closed curve C is equal to the area inside C?

Problem 12. A solid hemisphere of radius 1 has its lower flat base on the xy-plane and center at the origin. Its density function is $\delta = z$. Find the force of gravitational attraction it exerts on a unit mass at the origin.

Problem 13. Evaluate $\int_C (y-x)dz + (y-z)dz$ over the line segment C from $P : (1,1,1)$ to $Q : (2,4,8)$.

Problem 14. a) Let $\mathbf{F} = ay^2\mathbf{i} + 2y(x+z)\mathbf{j} + (by^2 + z^2)\mathbf{k}$. For what values of the constants a and b will F be conservative? Show work.

b) Using these values, find a function $f(x,y,z)$ such that $\mathbf{F} = \nabla f$.

c) Using these values, give the equation of a surface S having the property : $\int_P^Q \mathbf{F} \cdot dr = 0$ for any two points P and Q on the surface S.

Problem 15. Let S be the closed surface whose bottom face B is the unit disc in the xy-plane and whose upper surface is the paraboloid $z = 1 - x^2 - y^2$, $z \geq 0$. Find the flux of $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ across U by using the divergence theorem.

Problem 16. Using the data of the preceding problem, calculate the flux of $\mathbf{F}$ across U directly, by setting up the surface integral for the flux and evaluating the resulting double integral in the xy-plane.

Problem 17. An xz-cylinder in 3-space is a surface given by an equation $f(x,z) = 0$ in x and z alone; its section by any plane $y = c$ perpendicular to the y-axis is always the same xz-curve.

Show that if $\mathbf{F} = z^2\mathbf{i} + y^2\mathbf{j} + xz\mathbf{k}$ then $\oint_C \mathbf{F} \cdot dr = 0$ for any simple closed curve C lying on an xz-cylinder. (Use Stokes' theorem)

Problem 18. $\int e^{-x^2} dx$ is not elementary but $I = \int_0^\infty e^{-x^2} dx$ can still be evaluated.

a) Evaluate the iterated integral $\int_0^\infty \int_0^\infty e^{-x^2} e^{-y^2}\,dydx$, in terms of I.

b) Then evaluate the integral in (a) by switching to polar coordinates. Comparing the two evaluations, what value do you get for I?