Welcome back to recitation. In this video, I'd like us to work on the following problem. Which is we begin with a vector field, capital F, that is z*x*i plus z*y*j plus x*k. And we're going to look at the curve C that is a helix, that we can describe by the parameter t. So we'll describe it as cosine t comma sine t comma t. And we're interested in the portion of the helix that goes from (1, 0, 0) to (minus 1, 0, pi).

And I'd like you to do two things with this problem. The first thing I'd like you to do is I'd like you to sketch the curve that is carved out when you follow the t values that will start you at (1, 0, 0) and will finish you up at (minus 1, 0, pi). The second thing I would like you to do is I would like you to compute the line integral F dot dr over that portion of the helix. So there are two parts to this problem. Why don't you pause the video, work on these two parts, and then when you're feeling comfortable seeing the solution, bring the video back up and I'll show you how I did it.

OK, welcome back. So again what we're interested in doing in this problem is first, understanding what the curve looks like that we want to take this line integral over. And then actually computing the line integral. So we have this C that is a helix and it's described by cosine t, sine t, t. And in fact-- well, I won't say any more about this helix. But it actually should remind you when you see the picture of some portion of what DNA looks like. It's going to spiral around, just the way some little side of-- if you take DNA, how it spirals up, it's going to be the boundary of some of that. So we'll see that momentarily.

And if you notice, the first thing that's helpful if you want to sketch the curve, is that t-- I know immediately what the parameters are in t. t is ranging from 0 to pi. So I know already exactly what I want to do. And in order to draw this curve, what I'm going to do is I'm going to give myself a frame of reference. Because otherwise it's going to be really hard to draw this curve. And the frame of reference is the following.

All of these points, all of the points cosine t, sine t, t, lie-- in the x-y distance from the origin, they lie at a radius 1. So in terms of x and y, they're all going to sit on the boundary of a cylinder of radius 1. So let me draw what I mean by that and then we'll see where the points are. So let me actually come write right over here.

So my first part sketching the curve, the first thing I'm going to do is give myself a cylinder. Which I'll show you is of radius 1 momentarily. And I'm going to actually say this is the z-axis, coming straight through the middle. And then the y-axis is going to come out the side, as usual. And the x-axis is going to come down in this direction. OK, so this cylinder I'm thinking of, it has radius 1. So at any given z value, any fixed z value that I intersect with the cylinder gives a circle of radius 1.

Now if you notice again, what I was trying to explain from over here, is that if I don't look at the z component,

obviously the cosine t, sine t is something that's on the unit circle if I ignore the z component. And so that's how I know that these x and y values here are just going to lie on the cylinder, because they're always distance 1 from the z-axis, at any given height. So that's the first thing I observe.

The second thing I observe is that I mentioned t goes from 0 to pi, and that's exactly the z values also. So the z values are going from 0 to pi, so I know my first value is going to be on the xy-plane. And my last value is going to be on the z equals pi plane. And then the last thing to observe is that what is being carved out in the x and y components? Well, it's really exactly what you would do if you were trying to parameterize a circle. But you're parameterizing a circle and instead of just drawing it always on the same z plane-- so xy-plane, z equals a constant-- you're parameterizing that circle and you're also moving up.

And so the first value (1, 0, 0) is happening somewhere here. And the last value is happening at negative 1, 0, pi. And so I have to go sort of in the backwards direction. So here's my x equals negative 1, 0, pi. And so I'm kind of behind. You should think of this as being-- let me see if I can draw this-- this is behind the pi-- maybe I should draw it a little further. If this is the pi height, it was more like right there. It's behind the z-axis here. It's on the other side. And so the curve that's carved out is going to look something like this. It's going to come up through here. It's spiraling. And then it's going to go behind, on the other side of the cylinder, and spiral up.

Now in a perfect world, if I could draw this actually in three dimensions, the way it's coming up is actually, it has to have some sort of constant rate. Because it's always moving in the z direction at a constant rate. It's moving in the z direction linearly. Maybe this picture is not the most perfect picture because it looks like it's going up really fast at the end. But it gives us a feel for how the curve looks.

If I continued it, it would come back around to the front by the time t went to 2*pi. And so this is a spiral. It goes around the cylinder, behind the cylinder. And then if I go for another pi, from pi to 2*pi, it's going to go-- continue to curve around and then come back out to the front and be right above this point. So that's the helix. That's the shape of the helix. So this is an approximate sketch. Good thing I said sketch the curve. So this is a sketch of the curve.

And now what we want to do, again as I mentioned, is we want to compute a line integral, we want to compute F dot dr over this curve. So what I'm going to need is I'm going to know that this is the curve here, and I need to understand how to parameterize F and dr in terms of this parameter t. So that's what I'm interested in.

So let's think about what F dot dr is in order to do this. I think the notation you've seen from lecture is F we usually denote by capital P, capital Q, capital R. And dr we denote [dx, dy, dz]. And so F dot dr, as we've seen previously, is P*dx plus Q*dy plus R*dz. So let's see what that is in the parameters we have. So let me first remind ourselves, x in this situation is cosine t. y is sine t. And z is equal to t. Based on how we're parameterizing the curve. And

we're interested in the values of t going from 0 to pi. So these are the quantities that we're going to need.

Now in order to solve this problem I need dx, dy, dz. I also need P, Q, and R in terms of these x, y, and z values. So let me remind you also-- I'm going to write it over here so I don't have to look again-- F is actually z*x comma z*y comma x. Let me check that. Yes, that is what I have. So that is my F. So now that I have it a little closer, I'm going to put it all together. And I actually have to move over to put it together, but now the reference is closer.

So notice first I'm going to do P*dx. So P is z times x. And then what is dx? Well, dx-- I really need to figure out what dx/dt is, right? dx/dt is negative sine t. And so I'm going to replace the dx by a negative sine t dt. I'll be replacing dy by cosine t dt and I'll be replacing dz by dt. So I have all the pieces here; I just have to put them together. So let me do that.

I want to integrate over C F dot dr. I'm really going to be integrating-- let's see. Let me come to this side so I can see everything I need. I'm really going to be integrating-- I know I'm integrating from 0 to pi in my parameter t. My first component of F I told you was z*x. So that's t cosine t, and then dx is, as I said, negative sine t dt. So times negative sine t dt. That's my first component.

My next component, as I said, is Q*dy. Q was z*y and so it's t sine t. And then dy is cosine t dt. Oops, let me put the dt there, it's a little easier. And then I'm going to write the last component here, so we can see it all in one frame. And then the last thing was R*dz. And R is just x. So that's cosine t. And dz is just dt. So there's three components. This was the P*dx component, this is the Q*dy component, and this is the R*dz component.

And so notice, this is great. This is why I like this problem, it's going to be nice. Because I've got a t cosine t times negative sine t, and a t sine t times a cosine t. And so these two add up to 0, and so I only have to integrate one thing. So I only have to integrate from 0 to pi cosine t dt. And so what do I get? I get-- this should be sine t evaluated at 0 and pi. And sine of pi I believe is 0. And sine of 0 I believe is 0. And so I get 0 minus 0, so I get 0. So they're actually-- when I compute the line integral of F dot dr over that helix, I actually get 0.

So let me just remind you, real quick, what the point of the problem was and what we did. We had-- at the very beginning, we had a vector field, we had a curve, and essentially all we were doing is a problem we've done in two dimensions many times, which is compute a line integral along a curve. And so we just added a dimension. The problem is exactly the same. Instead of now just dx and dy, now we have a dx, dy, and a dz. We have one extra direction you're moving. But that's all that's different.

So the first thing we did was sketch the curve. Then we computed the line integral, as I'm saying, by exactly the same methods that we did in two dimensions. So everything, really, should remind you of what you've done previously, we now just have a third component we have to deal with. And that was, in our case, in this problem,

the R*dz part. So I think that's where I'll stop.