Problems: Vector Fields

1. Sketch the following vector fields. Pay attention to their names because we will be encountering these fields frequently.

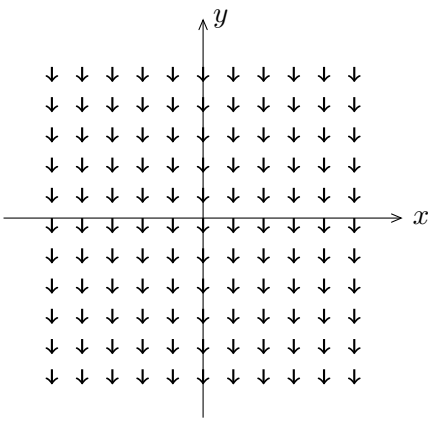
(a) Force, constant gravitational field $\mathbf{F}(x, y) = -g\mathbf{j}$.

(b) Velocity $\mathbf{V}(x, y) = \dfrac{x}{x^2 + y^2}\mathbf{i} + \dfrac{y}{x^2 + y^2}\mathbf{j} = \langle x, y\rangle / r^2$. (This is a shrinking radial field –like water pouring from a source at (0,0).)

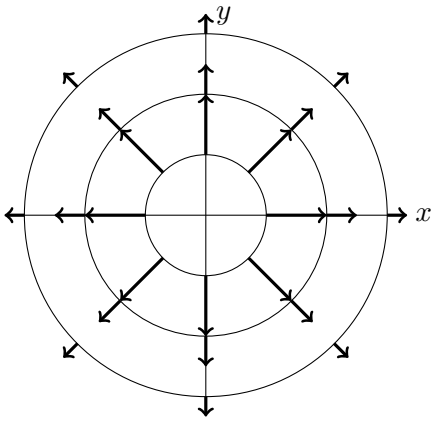
(c) Unit tangential field $\mathbf{F} = \langle -y, x\rangle / r$.

(d) Gradient $\mathbf{F} = \boldsymbol{\nabla} f$, where $f(x, y) = \dfrac{xy}{3}$ and $\boldsymbol{\nabla} f = \left\langle \dfrac{y}{3}, \dfrac{x}{3} \right\rangle$.

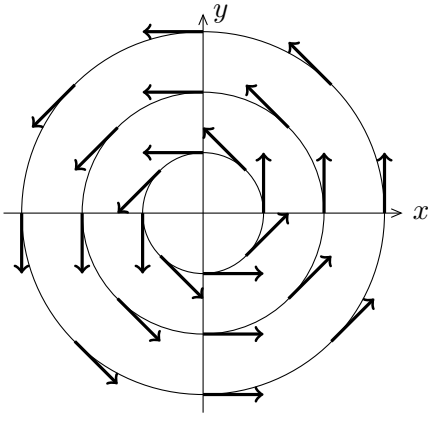
Answer: We visualize vector fields by drawing little arrows in the plane whose length and direction correspond to the magnitude and direction of the vector field at the base of the arrow.



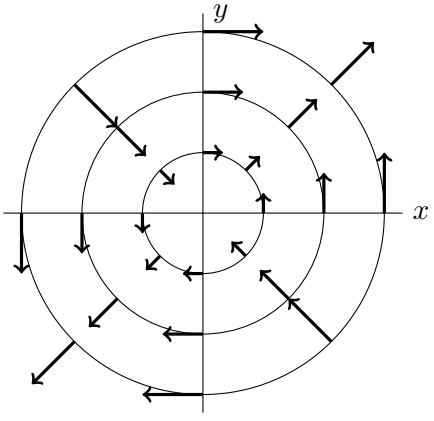
(a) Constant vector field



(b) Shrinking radial field



(c) Unit tangential field



(d) Gradient field $\boldsymbol{\nabla} f = \langle \frac{y}{3}, \frac{x}{3} \rangle$.

2. Compute the gradient field of $f(x, y) = xy^2$.

$$\begin{aligned}\boldsymbol{\nabla} f(x,y) &= \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} \\ &= y^2\,\mathbf{i} + 2xy\,\mathbf{j}.\end{aligned}$$

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