Meaning of Matrix Multiplication

1. In this problem we will show that multiplication by the matrix

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

acts by rotating vectors $45°$ counterclockwise. As usual, we write the vector $\mathbf{v} = x\mathbf{i} + y\mathbf{j}$ as a column vector $\begin{pmatrix} x \\ y \end{pmatrix}$.

a) Show that the length of $A\mathbf{v}$ is the same as the length of $\mathbf{v}$.

b) Use the dot product to show the angle between $\mathbf{v}$ and $A\mathbf{v}$ is $\pi/4$ radians.

c) Use the cross product to show $A\mathbf{v}$ is $\pi/4$ radians counterclockwise from $\mathbf{v}$.

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