

THIRD MIDTERM
MATH 18.022, MIT, AUTUMN 10

You have 50 minutes. This test is closed book, closed notes, no calculators.

Name: _____

Signature: _____

Recitation Time: _____

There are 5 problems, and the total number of points is 100. Show all your work. *Please make your work as clear and easy to follow as possible.*

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (20pts) For what values of λ does the function $f: \mathbb{R}^3 \rightarrow \mathbb{R}$,

$$f(x, y, z) = \lambda x^2 - \lambda xy + y^2 + \lambda z^2,$$

have a non-degenerate local minimum at $(0, 0, 0)$?

2. (20pts) Let $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ be the function $f(x, y, z) = x^2 - y^2 + z^2$.
(i) Show that f has a global maximum on the ellipsoid $2x^2 + 3y^2 + z^2 = 6$.

(ii) Find this maximum.

3. (20pts)

(i) Switch the order of integration in the integral

$$\int_0^3 \int_{x^2}^9 x e^{-y^2} dy dx.$$

(ii) Evaluate this integral.

4. (20pts) Let W be the region inside the sphere $x^2 + y^2 + z^2 = 1$ and inside the cone $z^2 = x^2 + y^2$. Set up an integral to calculate the integral of the function yz over W and calculate this integral.

5. (20pts) Let D be the region in the first quadrant bounded by the curves $x^2 - y^2 = 1$, $x^2 - y^2 = 4$, $xy = 1$ and $xy = 3$.

(i) Find $du dv$ in terms of $dx dy$, where $u = x^2 - y^2$ and $v = xy$.

(ii) Evaluate the integral

$$\iint_D (x^4 - y^4) dx dy.$$

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18.022 Calculus of Several Variables
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