

FIRST MIDTERM
MATH 18.022, MIT, AUTUMN 10

You have 50 minutes. This test is closed book, closed notes, no calculators.

Name: _____

Signature: _____

Recitation Time: _____

There are 5 problems, and the total number of points is 100. Show all your work. *Please make your work as clear and easy to follow as possible.*

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (20pts) (i) Suppose that the four vectors \vec{t} , \vec{u} , \vec{v} and \vec{w} lie in the same plane Π . Show that

$$(\vec{t} \times \vec{u}) \times (\vec{v} \times \vec{w}) = \vec{0}.$$

(ii) Now suppose that \vec{t} , \vec{u} , \vec{v} and \vec{w} are four non-zero vectors in \mathbb{R}^3 , such that

$$(\vec{t} \times \vec{u}) \times (\vec{v} \times \vec{w}) = \vec{0}.$$

Is it true that these four vectors have to lie in the same plane? If true, explain why and if false, give a counterexample.

2. (20pts) (i) Find a parametric equation for the line l through the two points $P = (1, -1, 2)$ and $Q = (-1, 3, 3)$.

(ii) Find the distance between the line l and the line m given parametrically by $(x, y, z) = (t - 1, 2t + 1, 3 - t)$.

3. (20pts) (i) Find the volume of the parallelepiped spanned by the vectors $\vec{u} = (1, 2, -3)$, $\vec{v} = (1, -2, 1)$ and $\vec{w} = (-1, -2, -1)$.

(ii) Do the vectors \vec{u} , \vec{v} and \vec{w} form a right-handed set or a left-handed set?

4. (20pts) Let D be the region inside the sphere of radius $2a$ centred at the origin and outside the cylinder of radius a centred around the z -axis.

(i) Describe the region D in cylindrical coordinates.

(ii) Describe the region D in spherical coordinates.

5. (20pts) Determine whether or not the following limits exist, and if they do exist, then find the limit. Explain your answer.

(i) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$.

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18.022 Calculus of Several Variables
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