Comparing Growth of $ln(x)$ and $x^{\frac{1}{3}}$

We have one more item on our original list of limits to cover; again we'll look at a slight variation on the original problem. We're going to find:

$$\lim_{x\to\infty} \frac{\ln x}{x^{1/3}}.$$

This limit is of the form $\frac{\infty}{\infty}$, so we apply l'Hôpital's rule to find:

$$\begin{aligned} \lim_{x\to\infty} \frac{\ln x}{x^{1/3}} &= \lim_{x\to\infty} \frac{1/x}{\frac{1}{3}x^{-2/3}} \qquad \text{(l'Hop)} \\ &= \lim_{x\to\infty} 3x^{-1/3} \\ &= 0 \end{aligned}$$

We conclude that $\ln x$ grows more slowly as x approaches infinity than $x^{1/3}$ or any positive power of x. In other words, $\ln x$ increases very slowly.

Question: When we discussed extensions of l'Hôpital's rule, we learned that we're allowed to change some hypotheses. How many hypotheses can we change at once?

Answer: We can make any or all of the three changes listed. However, $\dfrac{f(a)}{g(a)}$ must always be of the form $\frac{\infty}{\infty}$, $-\frac{\infty}{\infty}$, or $\frac{0}{0}$.