Extensions of L'Hôpital's Rule

Our first version of l'Hôpital's rule told us that:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

provided that $f(a) = g(a) = 0$ and $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ exists.

We've seen that we can get nearly equivalent results by replacing $f(x)$ and $g(x)$ by linear or quadratic approximations. L'Hôpital's rule is superior to the method of approximation because it works better in some situations.

It turns out that l'Hôpital's rule works even under the following conditions:

- $a = \pm\infty$

- $f(a), g(a) = \pm\infty$

- $\lim_{x \to a} \frac{f'(a)}{g'(a)} = \pm\infty$

In other words, l'Hôpital's rule works not just in the $\frac{0}{0}$ case but also when you're taking a limit of the form $\frac{\infty}{\infty}$. It will give us the right answer if $\frac{f'(a)}{g'(a)}$ approaches $-\infty$, ∞ or some finite number. It fails if $\frac{f'(a)}{g'(a)}$ oscillates wildly, but we don't encounter those conditions in this class; l'Hôpital's rule handles everything we could expect it to handle, and it's easy to use.

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