Example: $\int \sin^3 x \cos^2 x dx$

One of the exponents is odd so this is an easy, but not as easy as the previous example. We turn this integral into one in which the odd exponent is 1 by using the trig identity $\sin^x x + \cos^2 x = 1$ to remove the largest even power in the term with the odd exponent.

In this case the odd exponent is on $\sin x$, so we use:

$$\sin^2 x = 1 - \cos^2 x.$$

$$\begin{aligned} \int \sin^3 x \cos^2 x dx &= \int (\sin^2 x \cdot \sin x) \cos^2 x dx \\ &= \int (1 - \cos^2 x) \cdot \sin x \cos^2 x dx \\ &= \int (\cos^2 - \cos^4 x) \sin x dx \end{aligned}$$

This is now very similar to our previous example. We use the substitution:

$$\begin{aligned} u &= \cos x \\ du &= -\sin x dx \end{aligned}$$

To get:

$$\begin{aligned} \int \sin^3 x \cos^2 x &= \int (u^2 - u^4 x) \cdot (-du) \\ &= \int \frac{-u^3}{3} + \frac{u^6}{5} + c \\ &= -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + c \end{aligned}$$

At this point it's a good idea to check your work by differentiating your answer and then applying trig identities to see that the result equals the original integrand.