PROFESSOR: Hi. Welcome back to recitation. We've been talking in lecture about antiderivatives. So I have here a problem for you. Just an exercise about computing an antiderivative.

So the question is to compute an antiderivative of this big fraction. So on top it's got x to the eighth plus 2x cubed minus x to the 2/3 minus 3, that whole thing over x squared.

So just a quick linguistic note about why I said an antiderivative instead of the antiderivative, and then I'll let you work on it a little. So an antiderivative. There are many functions whose derivative is this function. Right? So they all differ from each other by constants. So I would be happy with any one as an answer to this question. That's why I chose the word an antiderivative. So I'm looking for a function whose derivative is equal to this function. So why don't you take a couple minutes, work this out, come back and you can check your answer against my work.

OK, welcome back. So we were just talking about this antiderivative here. So one thing you'll notice about this function is that I've written this in a sort of a silly form. And it's probably a lot easier to get a feel for what this function is if you break this fraction apart into its several pieces. So for example, x to the eighth over x squared is just x-- So, well OK, so let me, this antiderivative that I'm interested in, antiderivative of x to the eighth plus 2x cubed minus x to the 2/3 minus 3, over x squared dx. So I've written this in a silly form and you can get it in a nicer form if you just realize that, you know, this is just a sum of powers of x that I've put over this silly common denominator.

So our life will be a little simpler if we write this out by splitting it up into the separate fractions. So if I do that, this is just equal to the antiderivative of well, x to the eighth over x squared. That's x to the sixth. And 2 x cubed over x squared is just x. So I have x-- sorry, it's 2x-- plus 2x.

Now, OK so x to the 2/3 over x squared. So that's x to the 2/3 minus 2. Which is x to the minus 4/3. And minus 3 over x squared, so OK, so we could write that as minus 3 over x squared, or maybe it's a little more convenient to write it as minus 3 x to the minus 2, dx. So far I haven't really done anything, you know. A little bit of algebra here.

OK, but now we know that we've seen a formula for antidifferentiating a single power of x. I mean we know how to differentiate a single power of x, and so to do an antiderivative is just

the inverse process. And we also know that when you have the derivative of a sum, it's the sum of derivatives. And so consequently, if you have the antiderivative of a sum, it's just the sum of the antiderivatives. So we can write this out into its constituent parts. So it's the antiderivative of x to the sixth dx plus-- now of course you don't have to do this. You could probably proceed just from this step onwards, or, but I don't see any harm in actually splitting it up myself.

So antiderivative of 2x*dx minus, OK, x to the minus 4/3 dx plus minus 3 x to the minus 2 dx.

So I've just split it up into a bunch of pieces. I guess this one I sort of pulled the minus sign out and this one I didn't. But you know, whatever. Either way. OK so now we just need to remember our formulas for taking the antiderivative of a power of x. So in order to that, so when you take a derivative, the power goes down by one. So if you take an antiderivative to the power will always go up by one.

So in this case you get, so you're going to get x to the seventh. And now when you differentiate x to the seventh, a 7 comes down in front, right? You get 7 x to the sixth. So in order to get just x to the sixth, we have to also divide by that 7 there. So x to the sixth, the antiderivative is x to the seventh over 7. 2x, so that's going to give us plus 2 x squared over 2. Or if you like, you could just recognize right away that 2x is the derivative of x squared.

Minus-- OK now we've got minus powers. Rather, negative powers, so that always is a little trickier to keep track of. So again, the same thing is true though. You have to, you add one to the exponent. The exponent goes up by one when you take an antiderivative. It goes down by one when you take a derivative. So when you add 1 to minus 4/3 you get minus 1/3. So we have x to the minus 1/3. And now I have to divide by minus 1/3. When I take a derivative here, we get-- of x to the minus 1/3, I get minus 1/3 x to the minus 4/3. So I need to divide by that minus 1/3. OK.

And finally here so minus 3 x to the minus 2. So OK, so just like this first one, you might recognize that right off as the derivative of x to the minus 3. So this is plus-- Oh! Ha ha! You could do that if you were completely confused like me. So right, so x to the minus 2, it increased by one. Increases by 1. So when it increases by 1, you get minus 1 not minus 3. Oh! OK, good.

So this is minus 3 times x to the minus 1 over minus 1. OK. That's much better. And if you like, right, so, OK, so we could-- any constant we add to this, it'll still be an antiderivative. And now

we can do a little bit of arithmetic to arrange this into nicer forms if you wanted. So you could rewrite this as say, x to the seventh over 7 plus x squared plus 3 x to the minus 1/3 plus 3 x to the minus 1 plus a constant.

Now, suppose you got here and suppose that you did the same mistake that I just made. And you had accidentally thought that this was going to be a minus 1/3 power instead of a minus first power. So how would you, is there any way that you can prevent yourself making that mistake? Well there actually is. So one nice thing about antiderivatives is that it's really easy to check your work. After you've computed an antiderivative, or something that you think is antiderivative, you can always go back and take the derivative of the thing that you've computed and check that it's equal to what you started with. So if you, if you're ever worried that you made a mistake computing an antiderivative, one thing you can always do is take a derivative of what you've got at the end.

So if we take a derivative here we get x to the sixth plus 2x minus x to the minus 4/3 minus 3 x to the minus 2. OK? So that was just using our rule for powers one by one. And OK, so you say that out loud or write it down and then you just check. Right? So I said that, so that's exactly the same thing we've got right here. Yeah? So x to the sixth plus 2x minus x to the minus 4/3 minus 3x to the minus 2.

So one of the nicest things about antiderivatives, they can be difficult to figure out in the first place, but after you've got something that you think is antiderivative it's very easy to go back and check whether you did it correctly by taking the derivative and making sure that it matches the thing that you were trying to antidifferentiate at the beginning. So that's that.