CHRISTINE BREINER: Welcome to recitation.

Today what I'd like us to do is look at the inequality tangent x bigger than x for the x-values between 0 and pi over 2. We want to show that that is definitely true using the mean value theorem. And I want to point out that this was actually something that Joel used when he was a graphing tangent x and arctangent x on the same xy-plane. He used this fact in order to get the right-looking graph.

So what I'd like you to do, again, is I want you to show that for any x between 0 and pi over 2, tangent x is bigger than x. And use the mean value theorem to do it. I'll give you a little time to think about it, to work on it, and then I'll be back and we'll do it together.

OK. Welcome back.

So I'm going to take us through how to do this problem. And what I want to do is I want to point out a few things initially. So what I'm going to do is I'm going to remind us of the form of the mean value theorem that we need. And the form that we need will be f of x is equal to f of a plus f prime of c times x minus a. And here, remember, c has to be between a and x.

So in this case, what we want to consider is the region from 0 up to some x-value. So always, this is, if you think about it, the a and the x, well, a will be 0 and x will be the right-hand region, always less than pi over 2.

So what we need to check is, I'm going to consider f of x equal tangent x. And what I need to consider is, does tangent x satisfy the hypotheses of the mean value theorem on the region of interest? And so our region of interest will always be a equals 0 and b is equal to x, which is less than some pi over 2.

And it is true, tangent x is continuous between 0 and any value less than pi over 2. And it's also differentiable between 0 and any value less than pi over 2. So I can apply the mean value theorem to tangent x.

So in order to do this now, what I'd like to see is what kind of things I need for the right-hand side of this equation. So I obviously need to know what the output is at a, which is equal to 0. So let's recall f of 0. Well, tangent 0 is 0-- it's sine of 0 divided by cosine of 0. And then I need to evaluate the derivative somewhere between a, which is 0, and pi over 2. x can be any value less than pi over 2.

So let's evaluate what the derivative is in terms of x. We did this, actually, in another recitation. The derivative of tangent x is secant squared x.

And so now let's plug in what we know and then see what else we need to do in order to finish solving this problem.

OK. So what we have. f of x, I'm actually going to write tangent x, so we can see what's happening here. f of x is tangent x, and then I have that that equals, well, f of a is 0, plus f prime evaluated at c-- so that's going to be secant squared c-- times x minus a. Well, a here is 0, so this is just times x.

So we're very close. We're very close to showing tangent x is always bigger than x. In fact, you can see very easily what ultimately we need to show. We just need to show that secant squared c is bigger than 1 in our region of interest. And that would do it, because then this would be bigger than 1 times x. The right-hand side

would be bigger than 1 times x, so that would be sufficient.

So let's make sure we understand what secant squared c can look like. Or what the values can be. So we have c is between 0 and x, which is less than pi over 2. Right? That's where c is. So we always need to remember what values c could possibly have.

And then let's think about what we know about secant of c, there. Well, secant of c is equal to 1 over cosine of c. So if you can't remember what secant's values are, think about the values of cosine in that region. So I'm going to draw a rough sketch of the value of cosine between 0 and pi over 2. The value of cosine does something like this. This is 0, this is pi over 2. This is a very rough sketch. But this output is 1.

So between 0 and pi over 2, cosine of c is always less than 1. So 1 over cosine of c is always bigger than 1. OK? Because we're taking that reciprocal value. So again, cosine of c from 0 to pi over 2, not including 0, is always strictly less than 1. And so 1 over cosine c is always strictly greater than 1.

And so now I have the information I need to come back and finish the problem. So again, we were looking at the expression, we have tangent x is equal to secant squared c times x. Well, now I know secant c is bigger than 1, so now I know this whole thing is bigger than 1 times x, which is just equal to x.

So if you noticed, on the left-hand side we have a tangent x equals something which is bigger than x. So we've just shown that, for any value of x between 0 and pi over 2, tangent x is bigger than x.

And I think we'll stop there.