PROFESSOR: Hi everybody. Welcome back to recitation.

In lecture you did a bunch of examples of related rates problems. So I have a couple more for you to do today.

So here we've got-- OK, so we've got air being blown into a spherical balloon at a rate of 1,000 cubic centimeters per second. So the question is, how fast is the radius growing when the radius is equal to 8 centimeters? And then OK, so I've got a second question, which is, how fast is the surface area growing at that same time?

So why don't you take a few minutes, work this one out for yourself, come back, and we'll work it out together.

All right. Welcome back.

So this question, like all related rates questions, has the property that the calculus is typically very straightforward, but that there's some geometric or algebraic setup. So in this case, it's straight up geometry.

So we have a balloon, we know it's a perfect sphere, we know how fast the volume is changing. So OK, but we need to know how fast the radius is changing. So in order to do that we need to figure out a relationship between the radius and the volume. And then we can just do implicit differentiation like we've been doing.

So for example, OK, so for a sphere-- the setup is not so bad for this first one-- so we know that for a sphere the volume is equal to 4/3 pi times the radius cubed. So that's the fundamental relationship between the volume and radius of a sphere, and it's true for every sphere everywhere in Euclidean space.

And we're given also, that the volume is changing at a constant rate of 1,000 centimeters cubed per second. So dV/dt is just given to be 1,000. You know, leave off the units at this point.

So the question is, what is dr/dt? That's what we're trying to figure out. We're trying to figure out how fast the radius is changing at the moment when the radius is equal to 8 centimeters. So how can we do that? Well, this fundamental relationship, it's an identity. It always holds. So that means we can differentiate it.

So if we take the derivative of this identity, well, V on the left just becomes dV/dt. And on the right we want to do implicit differentiation. So here r is changing with respect to time. r is a function of t. Well, OK. So 4/3 pi is a constant. So that when we differentiate nothing happens. 4/3 pi. And so now we differentiate r cubed with respect to t, so that gives us 3 r squared times dr/dt. So that's just the chain rule in action there.

And now what we want is this dr/dt. Right? That's the thing that we're looking for, is how fast the radius is growing. So that's dr/dt. And we want it at the moment when r is equal to 8. So when r is equal to 8, this implies that-- well, OK, so dV/dt is 1,000 always. And it implies that-- OK, so it's equal to, 1,000 is equal to 4/3 pi times 3 times 8 squared times dr/dt. So at this moment that we're interested in, we have this equation to solve, for dr/dt, and this is a nice, simple equation to solve. You just divide through by everything on the right-hand side other than dr/dt. So this implies that dr/dt is equal to-- well, OK, so I have to divide 1,000 by all this stuff. I think it works out to something like 125 over 32*pi. All right.

So that's the exact value. Maybe you're interested in sort of knowing about how large this is. So 32*pi is pretty close to 100, so this is about 1.2 something.

So, all right. So there we go. So that answers the first question. At that moment the radius is growing at a rate of 125 over 32*pi centimeters per second.

OK. So now how about the second question that we've got here? What about the surface area? So again, we know how fast now the radius is changing and we know how fast the volume is changing. So in order to figure out how fast the surface area is changing, we need something that relates the surface area to either the volume or the radius. Now the relationship between surface area and volume is something that we could sort of work out if we had to, but it's a lot easier to write down the surface area in terms of the radius. So let's do that.

So we have, I'm going to use the letter S to denote surface area of a sphere. So again, it's a general identity, you know, a geometric fact that the surface area of a sphere is equal to 4*pi times the radius squared. And this is always true. And now the thing that we want is the rate of change of the surface area. So the rate of change is the derivative. So we want to compute the derivative here, dS/dt.

So OK, so we just do it. So dS/dt is equal to-- well, 4*pi hangs around. And again, we differentiate r squared. r is a function of t, so we have to use the chain rule here. So this is times 2r times dr/dt. So this is an identity. So this is true always.

And now we want to know, at this particular moment in time, when r is equal to 8, what is ds/dt? And in order figure that out, well OK, we just have to be able to plug in for everything else. So when r equals 8-- all right, well luckily, you know, if we were just starting this problem from scratch here, we'd have a problem. Which is we wouldn't know what dr/dt was. But luckily, we've already figured it out, right? In the first part of the problem. So we know that when r is equal to 8, dr/dt is equal to 125 over 32*pi. Did I copy that right? Yes, I did. OK.

So OK, so in this case, the equation we have to solve is just completely straightforward. We just plug in the values and it's already solved for us. So that's nice. So that we get that ds/dt at that moment is equal to-- well, it's 4*pi times 2 times 8 is the radius times 125 over 32*pi. Oh boy, and all right. So we can work this out if we want, I guess. That's 32*pi to cancel, so that's equal to 250. And I guess the units there better be centimeters squared per second.

So at this moment in time the surface area is growing by 250 centimeters squared per second. So that's all we had to do.

So we're all set.