Approximations at 0 for Sine, Cosine and Exponential Functions

Here is a list of several linear approximations which you may want to memorize. Half the work of memorizing a linear approximation is memorizing the derivative of a function at a base point, so memorizing these formulas should improve your knowledge of derivatives.

To make things as simple as possible, we always use base point $x_0 = 0$ and assume that $x \approx 0$. Then our general formula becomes:

$$f(x) \approx f(0) + f'(0)x.$$

Remember that when x is not near zero, this approximation probably won't work.

(Later we'll discuss exactly how close x has to be to zero; this is partly a matter of intuition and is very important in applications.)

We want to find linear approximations for the functions $\sin x$, $\cos x$ and e^x when x is near 0. We'll start by building a table of values of $f'(x)$, $f(0)$, and $f'(0)$; from these we can "read off" the linear approximations.

$f(x)$	$f'(x)$	$f(0)$	$f'(0)$
$\sin x$	$\cos x$	0	1
$\cos x$	$-\sin x$	1	0
e^x	e^x	1	1

We can now plug the values for $f(0)$ and $f'(0)$ into our formula $f(x) \approx f(0) + f'(0)x$ to get linear approximations for these functions:

1. $\sin x \approx x$ (if $x \approx 0$)   (see part (a) of Fig. 1)

2. $\cos x \approx 1$ (if $x \approx 0$)   (see part (b) of Fig. 1)

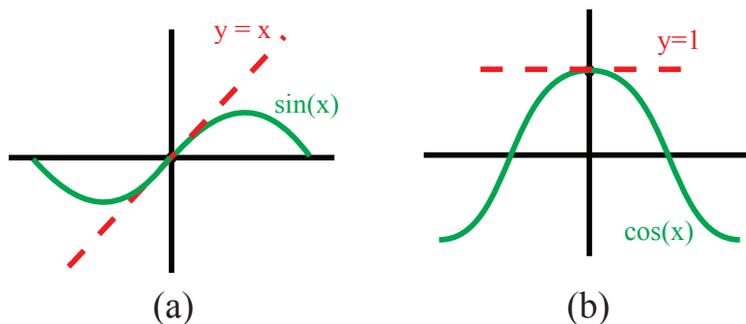3. $e^x \approx 1 + x$ (if $x \approx 0$)

Figure 1: Linear approximations to sine and cosine at $x = 0$.

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