PROFESSOR: Hi. Welcome back to recitation. Last time in lecture you started learning about implicit differentiation. And you saw some examples of how implicit differentiation can be used to compute derivatives of functions defined implicitly. So let's do another example today.

So here I have a curve that's defined by the implicit equation y cubed plus x cubed equals 3x*y. And I'd like to know what the slope of the tangent line to that curve is at the point (4/3, 2/3).

So before we start doing anything, let me just make a couple of observations. If you don't believe me, that the point (4/3, 2/3) is on this curve, you can always check by plugging the values in and confirm that really, yes, 4/3 cubed plus 2/3 cubed is equal to 3 times 2/3 times 4/3.

So it's-- how I found this point is maybe a little magical. Because as you can see, this equation is really a tough one to solve for y. What you sort of-- natural thing to want to do when asked this question is to solve for y and get an equation for y in terms of x and then take the derivative using the various differentiation rules that you've learned. But here, this is-- I'll let you in a secret. There is a way to do this. But it's really hard. It's really ugly and it's beyond the scope of this course. So really we're much better off treating this as an implicit differentiation problem than as an explicit differentiation problem.

So having said that, why don't you take a minute or two. Try and have a go at this yourself. And then we'll come back and work through it together.

All right, so welcome back. We were in the middle-- we were just about to start, actually-- solving this problem, computing the slope of the tangent line to the curve y cubed plus x cubed equals 3x*y at this point, (4/3, 2/3). So the slope of the tangent line is the value y prime of x at that point. So we need to answer this question. What we need to do is we need to find the derivative of y. But as I said earlier, this is tough to do explicitly, to find y in terms of x, so we're going to use implicit differentiation.

So, so we start with this equation, y cubed plus x cubed equals 3x*y and we can take a derivative with respect to x. So some parts-- all right, so let's start with it in the order it's given. So you have y cubed. If you take a derivative of y cubed with respect to x, what you need to use the chain rule because y is implicitly a function of x and so y cubed is the chain rule. It's the cubed function applied to the y function. And this is true of implicit differentiation in general. That the reason that we can do this, the really fundamental reason this works is that we have the chain rule and that it lets us evaluate derivatives of compositions.

So in our case we have, so we take a derivative of the whole thing, of this whole-- I'm going to write, this is a little sloppy notation, but I hope you know what I mean. We have this identity and so we're going to take a derivative of the whole thing. And so the first part on the left, we get the derivative of y cubed. So by the chain rule, so we first take the derivative of the cube function at y and then multiply by the derivative of y. So this is the derivative of y cubed. It just gives us 3 y squared. So that's what happens when you just deal with the cubed part. But then we need to multiply by-- in the chain rule-- by the derivative of the inside. Which in this context is dy by dx. OK. Plus the derivative of x cubed. That's more straightforward. Nothing really complicated going on here. We've seen this for a little while now. It's just 3x squared equal--

OK. So on the right now, we don't actually have a chain rule, we have a product rule situation here. We have 3 times x times y. So 3 is just a constant. And so we could just pull it out in front. So we take the derivative of x*y. So we take the derivative of the first times the second plus the derivative of the second times the first. So the derivative of the first is just-- ah sorry, x is the first, so its derivative is 1. So we got 3 times the second is y. Plus-- OK, so we take the first times the derivative of the second, which is dy by dx.

So because this is an identity it holds for all values of x and y. This equality follows just by taking the derivative of both sides. Good. So now the thing we want is that we want the slope of the tangent line at a particular point. So we want to isolate dy/dx. That's the thing we're trying to find. So here, if you're only interested in dy/dx, this is actually a linear equation in some sense, right? We have dy/dx, a constant, something-- or, it's not a constant-- something times dy/dx plus something equals something plus something times dy/dx. There are no squares of dy/dx is what I really mean.

So OK. So that that's nice. It makes it relatively easier to solve, so we can just combine all the terms with dy/dx in them. Let's say we'll combine maybe, put them over here and put everything else over there. So over here we get, so dy/dx, so we have a 3 y squared minus a 3x. And on the other side we have a 3y minus a 3 x squared. And so this is times, multiplication there. And so we want dy by dx just by itself. So we can just divide through by 3 y squared minus 3x. So then we have dy/dx is equal to-- well, all right, so there are a lot of 3's here. There's a constant multiple of 3 on this side, a constant multiple of 3 on this side. Those are going to cancel.

So this is y minus x squared over y squared minus x. OK, so at any point (x, y) on this curve, the slope of the tangent line is given by this expression here. And we're interested in a particular point in this problem. We're interested in the point 4/3 comma 2/3. So at-- let me take that back up here-- so at the point 4/3 comma 2/3 we have dy by dx.

So OK, we just we just plug that value of y and that value of x into this formula that we've got. So that's 2/3 minus 4/3 squared is 16/9 over-- well, let's see, 2/3 squared is 4/9 minus 4/3. All right, so we have a little bit of rational number arithmetic here. Maybe I'll multiply top and bottom through by 9 to get 6 minus 16 over 4 minus 12. So this is negative 10 over negative 8, which is 5 over 4.

And if we go back to the picture that I drew, it actually looks pretty reasonable over here, right? This slope of this tangent line is actually a little bit bigger than 1. Great. So that's that.