Differentiation Formulas

General Differentiation Formulas

$$
\begin{aligned}
(u+v)' &= u' + v' \\
(cu)' &= cu' \\
(uv)' &= u'v + uv' \quad \text{(product rule)} \\
\left(\frac{u}{v}\right)' &= \frac{u'v - uv'}{v^2} \quad \text{(quotient rule)} \\
\frac{d}{dx} f(u(x)) &= f'(u(x)) \cdot u'(x) \quad \text{(chain rule)}
\end{aligned}
$$

Implicit differentiation

Let's say you want to find y' from an equation like

$$y^3 + 3xy^2 = 8$$

Instead of solving for y and then taking its derivative, just take $\frac{d}{dx}$ of the whole thing. In this example,

$$
\begin{aligned}
3y^2y' + 6xyy' + 3y^2 &= 0 \\
(3y^2 + 6xy)y' &= -3y^2 \\
y' &= \frac{-3y^2}{3y^2 + 6xy}
\end{aligned}
$$

Note that this formula for y' involves both x and y.

As we see later in this lecture, implicit differentiation can be very useful for taking the derivatives of inverse functions and for logarithmic differentiation.

Specific differentiation formulas

You will be responsible for knowing formulas for the derivatives of these functions:

$$x^n,\ \sin^{-1} x,\ \tan^{-1} x,\ \sin x,\ \cos x,\ \tan x,\ \sec x,\ e^x,\ \ln x.$$

You may also be asked to derive formulas for the derivatives of these functions.

For example, let's calculate $\frac{d}{dx} \sec x$:

$$\frac{d}{dx} \sec x = \frac{d}{dx} \frac{1}{\cos x} = \frac{-(-\sin x)}{\cos^2 x} = \tan x \sec x$$

You may be asked to find $\frac{d}{dx}\sin x$ or $\frac{d}{dx}\cos x$ using the following information:

$$\begin{aligned}
\lim_{h\to 0}\frac{\sin(h)}{h} &= 1\\
\lim_{h\to 0}\frac{\cos(h)-1}{h} &= 0
\end{aligned}$$