PROFESSOR: Today we're going to continue our discussion of methods of integration. The method that I'm going to describe today handles a whole class of functions of the following form. You take P(x) / Q(x) and this is known as a rational function. And all that means is that it's a ratio of two polynomials, which are these functions P(x) and Q(x). We'll handle all such functions by a method which is known as partial fractions. And what this does is, it splits P / Q into what you could call easier pieces.

So that's going to be some kind of algebra. And that's what we're going to spend most of our time doing today. I'll start with an example. And all of my examples will be illustrating more general methods. The example is to integrate the function 1 1/(x-1) plus, say, 3/(x+2), dx. That's easy to do. It's just, we already know the answer. It's ln x - ln(x-1) + 3 ln(x+3). Plus a constant. So that's done. So now, here's the difficulty that is going to arise. The difficulty is that I can start with this function, which is perfectly manageable. And than I can add these two functions together. The way I add fractions. So that's getting a common denominator. And so that gives me x+2 here, plus 3(x-1). And now if I combine together all of these terms, then altogether I have 4x + 2 - 3, that's -1. And if I multiply out the denominator that's x^2 plus-- That 2 turned into a 3, that's interesting. I hope there aren't too many more of those transformations. Is there another one here?

STUDENT: [INAUDIBLE]

PROFESSOR: Oh, it happened earlier on. Wow that's an interesting vibration there. OK. Thank you. So, I guess my 3's were speaking to my 2's. Somewhere in my past. OK, anyway, I think this is now correct. So the problem is the following. This is the problem with this. This integral was easy. I'm calling it easy, we already know how to do it. Over here. But now over here, it's disguised. It's the same function, but it's no longer clear how to integrate it. If you're faced with this one, you say, what am I supposed to do. And we have to get around that difficulty. And so what we're going to do is we're going to unwind this disguise. So we have the algebra problem that we have. Oh, wow. There must be something in the water. Impressive. Wow. OK, let's see. Is

2/3 = 3/2? Holy cow. Well that's good. Well, I'll keep you awake today with several other transpositions here.

So our algebra problem is to detect the easy pieces which are inside. And the method that we're going to use, the one that we'll emphasize anyway, is one algebraic trick which is a shortcut, which is called the cover-up method. But we're going to talk about even more general things than that. But anyway, this is where we're headed. Is something called the cover-up method. Alright. So that's our intro. And I'll just have to remember that 2 is not 3. I'll keep on repeating that.

So now here I'm going to describe to you how we unwind this disguise. The first step is, we write down the function we want to integrate. Which was this. And now we have to undo the first damage that we did. So the first step is to factor the denominator. And that factors, we happen to know the factors, so I'm not going to carry this out. But this can be a rather difficult step. But we're going to assume that it's done. For the purposes of illustration here. So I factor the denominator. And now, the second thing that I'm going to do is what I'm going to call the setup here. How I'm going to set things up. And I'll tell you what these things are more systematically in a second. And the setup is that I want to somehow detect what I did before. And I'm going to write some unknowns here.

What I expect is that this will break up into two pieces. One with the denominator x - 1, and the other with the denominator x + 2. So now, my third step is going to be to solve for A and B. And then I'm done, if I do that. That's the complete unwinding of this disguise. And this is where the cover-up method comes in handy. This is this method that I'm about to describe. Now, you can do the algebra in a clumsy way, or you can do it in a quick way. And we'd like to get efficient about the algebra involved. And so let me show you what the first step in the trick is. We're going to solve for A by multiplying by (x - 1). Now, notice if you multiply by (x - 1) in that equation 2, what you get is this. You got (4x - 2) divided by-- The (x-1)'s cancel. You get this on the left-hand side. And on the right-hand side you get A. The (x-1)'s cancel again. And then we get this extra term. Which is B x+2 times x-1. Now, the trick here, and we're going to get even better trick in just a second. The trick here is that I didn't try to clear the denominators completely. I was very efficient about the way I did it. It just cleared one factor.

And the result here is very useful. Namely, if I plug in now x = 1, this term drops out too. So what I'm going to do now is I'm going to plug in x = 1. And what I get on the left-hand side here is 4 - 1 and 1 + 2, and on the left-hand side I get A. That's the end. This is my formula for A. A happens to be equal to 1. And that's, of course, what I expect. A had better be 1, because the thing broke up into 1/(x-1) + 3/(x+2). So this is the correct answer. There was a question out here, which I missed.

STUDENT: Aren't polynomials defined as functions with whole powers, or could they be square roots?

PROFESSOR: Are polynomials defined as functions with whole powers, or can they be square roots? That's the question. The answer is, they only have whole powers. So for instance here I only have the power 1 and 0. Here I have the powers 2, 1 and 0 in the denominator. Square roots are no good for this method. Another question.

STUDENT: [INAUDIBLE]

PROFESSOR: Why did I say x = 1? The reason why I said x = 1 was that it works really fast. You can't know this in advance, that's part of the method. It just turns out to be the best thing to do. The fastest way of getting at the coefficient A. Now the curious thing, let me just pause for a second before I do it. If I had plugged x = 1 into the original equation, I would have gotten nonsense. Because I would've gotten 0 in the denominator. And that seems like the most horrible thing to do. The worst possible thing to do, is to set x = 1. On the other hand, what we did is a trick. We multiplied by x - 1. And that turned the equation into this. So now, in disguise, I multiplied by 0. But that turns out to be legitimate. Because really this equation is true for all x except 1. And then instead of taking x = 1, I can really take x tends to 1. That's really what I need. The limit as x goes to one. The equation is still valid then. So I'm using the worst case, the case that looks like it's dividing by 0. And it's helping me because it's cancelling out all the information in terms of B. So the advantage here is this cancellation that occurs in this part.

So that's the method. We're going to shorten it much, much more in a second. But let me carry it out for the other coefficient as well. So the other coefficient I'm going to solve for B, I'm going to multiply by x + 2. And when I do that, I get (4x-1)/(x-1), that's the left-hand side, the very top expression there. And then down below I get A/(x-1) ((x+2). And then again the (x+2)'s cancel. So I get B sitting alone. And now I'm going to do the same trick. I'm going to set x = -2. That's the value which is going to knock out this A term here. So that cancels this term completely. And what we get here all told is - 8 - (-8 - 1)/(-2 - 1) = B. In other words, B = 3, which was also what it was supposed to be. B was this number 3, right here. Which I'm now not going to change to 2. Because I know that it's not 2. There was a question.

STUDENT: [INAUDIBLE]

PROFESSOR: All right. Now, this is the method which is called cover-up. But it's really carried out much, much faster than this. So I'm going to review the method and I'm going to show you what it is in general. So the first step is to factor the denominator, Q. That's what I labeled 1 over there. That was the factorization of the denominator up top. The second step is what I'm going to call the setup. That's step 2. And that's where I knew what I was aiming for in advance. And I'm going to have to explain to you in every instance exactly what this setup should be. That is, what the unknowns should be and what target, simplified expression, we're aiming for. So that's the setup. And then the third step is what I'll now call cover-up. Which is just a very fast way of doing what I did on this last board, which is solving for the unknown coefficients. So now, let me perform it for you again. Over here. So it's 4x - 1 divided by-- So this is to eliminate writing here. Handwriting. It makes it much faster. So this part just factoring the denominator, that was 1, that was step 1. And then step 2, again, is the setup, which is setting it up like this. Alright, that's the setup. And now I claim that without writing very much, I can figure out what A and B are. Just by staring at this.

So now what I'm going to do is I'm just going to think what I did over there. And I'm just going to do it directly. So let me show you what the method consists of visually. I'm going to cover up, that is, knock out this factor, and focus on this number here. And I'm going to plug in the thing that makes this 0, which is x = 1. So I'm plugging in x = 1. To this left-hand side. And what I get is (4 - 1)/(1 + 2) = A. Now, that's the same thing I did over there. I just did it by skipping the intermediate algebra step, which is a lot of writing. So the cover-up method really amounts to the following thing. I'm thinking of multiplying this over here. It cancels this and it gets rid of everything else. And it just leaves me with A on the right-hand side. And I have to get rid of it on this side. So in other words, by eliminating this, I'm isolating a on the right-hand side. So the cover-up is that I'm covering this and getting A out of it.

Similarly, I can do the same thing with B. It's focused on the value x = -2. And B is what I'm getting on the right-hand side. And then I have to cover up this. So if I cover up that, then what's left over, with x = -2, is again (-8 - 1)/(-2 - 1). So this is the way the method gets carried out in practice. Writing, essentially, the least you can.

Now, when you get to several variables, it becomes just way more convenient to do this. So now, let me just review when cover-up works. So this cover-up method works if Q(x) has distinct linear factors. And, so you need two things here. It has to factor completely, the denominator has to factor completely. And the degree of the numerator has to be strictly less than the degree of the denominator. I'm going to give you an example here. So, for instance-- And this tells you the general pattern of the setup also. Say you had x^2 + 3x + 8, let's say. Over (x - 1) (x - 2) (x + 5). So here I'm going to tell you the setup.

The setup is going to be - A/(x-1) + B/(x-2) + C/(x+5). And it will always break up into something. So however many factors you have, you'll have to put in a term for each of those. And then you can find each number here by this cover-up method.

Now we're done with that. And now we have to go on to the algebraic complications. So would the first typical algebraic complication be. It would be repeated roots or repeated factors. Let me get one that doesn't come out to be extremely ugly here. So this is what we'll call Example 2. And this is going to work when the degree, you always need that the degree of the numerator is less than the degree of the denominator. And Q has now repeated linear factors. So let's see which example I wanted to show you. So let's just give this here. I'll just repeat the denominator. With an extra factor like this. Now, the main thing you need to know, since I've already performed the factorization for you. Already performed Step 1. This is Step 1 here. You have to factor things all the way, and that's already been done for you.

And here's what this setup is. The setup is that it's of the form A/(x-1) plus B/(x-1)^2 - we need another term for the square here - plus C/(x+2). In general, if you have more powers you just need to keep on putting in those powers. You need one for each of the powers. Why does it have to be squared? OK. Good question. So why in the world am I doing this? Let me just give you one hint as to why I'm doing this. It's very, very much like the decimal expansion of a number or, say, the base 2 expansion of a number. So, for, example the number 7/16 is 0/2 + 1/2^2 + 1/2^3 plus, is that right? So it's 4/16 + 1/2^4. It's this sort of thing. And I'm getting this power and this power. If I have higher powers, I'm going to have to have more and more. So this is what happens when I have a 2^4. I have to represent things like this. That's what's coming out of this piece with the repetitious here. Of the powers. This is just an analogy. Of what we're doing. Yeah, another question over here.

STUDENT: [INAUDIBLE]

PROFESSOR: Yes. So this is an example, but it's meant to represent the general case and I will also give you a general picture. For sure, once you have the second power here, you'll need both the first and the second power mentioned over here. And since there's only a first power over here I only have to mention a first power over there. If this were a 3 here, there would be one more term which would be the one for (x-1)^3 in the denominator. That's what you just said. OK, now, what's different about this setup is that the cover-up method, although it works, it doesn't work so well. It doesn't work quite as well.

The cover-up works for the coefficients B and C, not A. We'll have a quick method for the numbers B and C. To figure out what they are. But it will be a little slower to get to A, which we will do last. Let me show you how it works. First of all, I'm going to do the ordinary cover-up with C. So for C, I just want to do the same old thing that I did before. I cover up this, and that's going to get rid of all the junk except for the C term. So I have to plug in x = -2. And I get x-- sorry, I get (-2)^2 + 2 in the numerator. And I get (-2 - 1)^2 in the denominator. Remember I'm covering this up. So that's all there is on the left-hand side. And on the right-hand side all there is C. Everything else got killed off, because it was x - 2 times that. That's 0 times all that other stuff. And the x - 2 over here canceled. This is the shortcut that I just described, and this is much faster than doing all that arithmetic. And algebra. So all together this is a 6/9, right? So it's C = 6/9, which is 2/3.

Now, the other one which is easy to do, I'm going to do by the slow method first. But you omit a term. The idea is to cover up the other bad factor. Cover-ups, I'll do it both the fast way and the slow way. I'll do it the fast way first, and then I'll show you the slow way. The fast way is to cover this up. And then I have to cover up everything else. That gets eliminated. And that includes everything but B. So I get B on this side. And I get 1 on that side. So that's (1^2 + 2) / (1 + 2). So in other words, B = 1. That was pretty fast, so let me show you what arithmetic was hiding behind that. What algebra was hiding behind it. What I was really doing is this. And multiplying through by (x-1)^2, so I got this. So this canceled here, so this C just stands alone. And then I have here C/(x+2) (x-1)^2. Notice again, I cleared out that one-- this term from the denominator and sent it over to the other side. Now, what's happening is that when I set x = 1 here, this term is dying. This term is going away, because there's more powers in the numerator than in the denominator. This is still 0. And this one is gone also.

So all that's left is B. Now, I cannot pull that off with a single power of x - 1. I can't expose the A term. It's the B term that I can expose. Because I can multiply through by this thing squared. If I multiply through by just x - 1, what'll happen here is I won't have canceled this (x-1)^2. It's useless. I still have a 0 in the denominator. I'll have B / 0 when I plug in x = 1. Which I can't use. Again, the cover-up method is giving us B and C, not A.

Now, for the last term, for A, I'm going to just have to be straightforward about it. And so I'll just suggest for A, plug in your favorite number. So plug in my favorite number. Which is x = 0. And you won't be able to plug in x = 0 if you've already used it. Here the two numbers we've already used are x = 1 and x = -2. But we haven't used x = 0 yet, so that's good. I'm going to plug in now x = 0 into the equation. What do I get? I get 0 (0^2 + 2) / ((-1)^2 * 2) is equal to, let's see. A is the thing that I don't know. So it's A A/(-1) + B/(x-1)^2, so B = 1, so that's 1 1/(-1)^2. And then C was 2/3. 2/3 / (x+2). So that's 0 + 2. Don't give up at this point. This is a lot of algebra. You really have to plug in all these numbers. You make one arithmetic mistake and you're always going to get the wrong answer. This is very arithmetically intensive. However, it does simplify at this point. We have 2/2, that's 1. Is equal to -A + 1 + 1/3. So let's see. A on the other side, this becomes A = 1/3. And that's it. This is the end. We've we've simplified our function. And now it's easy to integrate. Question. Another question.

STUDENT: [INAUDIBLE]

PROFESSOR: So the question is, if x = 0 has already been used, what do I do? And the answer is, pick something else. And you said pick a random number. And that's right, except that if you really picked a random number it would be 4.12567843, which would be difficult. What you want to pick is the easiest possible number you can think of. Yeah.

STUDENT: [INAUDIBLE]

PROFESSOR: If you had, as in this sort of situation here, more powers. Wouldn't you have more variables. Very good question. That's absolutely right. This was a 3 by 3 system in disguise, for these three unknowns, A, B and C. What we started with in the previous problem was two variables. It's over here, the variables A and B. And as the degree of the denominator goes up, the number of variables goes up. It gets more and more and more complicated. More and more arithmetically intensive.

STUDENT: [INAUDIBLE]

PROFESSOR: Well, so. The question is, how would you solve it if you have two unknowns. That's exactly the point here. This is a system of simultaneous equations for unknowns. And we have little tricks for isolating single variables. Otherwise we're stuck with solving the whole system. And you'd have to solve the whole system by elimination, various other tricks. I'll say a little more about that later.

Now, I have to get one step more complicated with my next example. My next example is going to have a quadratic factor. So still I'm sticking to the degree of the polynomial and the numerator is less than the degree of the polynomial in the denominator. And I'm going to take the case where Q has a quadratic factor. Let me just again illustrate this by example. I have here (x - 1) (x^2 + 1). I'll make it about as easy as they come. Now, the setup will be slightly different here. Here's the setup. It's already factored. I've already done as much as I can do. I can't factor this x^2 + 1 into linear factors unless you know about complex numbers. If you know about complex numbers this method becomes much easier. And it comes back to the cover-up method. Which is the way that the cover-up method was originally conceived by Heaviside. But you won't get to that until 18.03. So we'll wait. This, by the way, is a method which is used for integration. But it was invented to do something with Laplace transforms and inversion of certain kinds of differential equations. By Heaviside. And so it came much later than integration. But anyway, it's a very convenient method.

So here's the setup with this one. Again, we want a term for this (x - 1) factor. And now we're going to also have a term with the denominator x^2 + 1. But this is the difference. It's now going to be a first degree polynomial. One degree down from the quadratic here. So this is what I keep on calling the setup, this is number 2. You have to know that in advance based on the pattern that you see on the left-hand side. Yes.

STUDENT: [INAUDIBLE]

PROFESSOR: The question is, if the degree of the numerator-- So in this case, if this were cubed, and this is matching with the denominator, which is total of degree 3. The answer is that this does not work.

STUDENT: [INAUDIBLE]

PROFESSOR: It definitely doesn't work. And we're going to have to do something totally different to handle it. Which turns out, fortunately, to be much easier than this. But we'll deal with that at the end. Keep this in mind. This is an easy way to make a mistake if you start with a higher degree numerator. You'll never get the right answer. So now, so I have my setup now. And now what can I do? Well, I claim that I can still do cover-up for A. It's the same idea. I cover this guy up. And if I really multiply by it it would knock everything out but A. So I cover this up and I plug in x = 1. So I get here 1^2 / (1^2 +1) = A. In other words, A = 1/2. Again cover-up is pretty fast, as you can see. It's not too bad.

Now, at this next stage, I want to find B and C. And the best idea is the slow way. Here, it's not too terrible. But it's just what we're going to do. Which is to clear the denominators completely. So for B and C, just clear the denominator. That means multiply through by that whole business. Now, when you do that on the left-hand side you're going to get x^2. Because I got rid of the whole denominator. On the right-hand side when I bring this up, the x - 1 will cancel with this. So the A term will be A(x^2 + 1). And the Bx + C term will have a remaining factor of x - 1. Because the x^2 + 1 will cancel. Again, the arithmetic here is not too terrible.

Now I'm going to do the following. I'm going to look at the x^2 term. On the left-hand side and the right-hand side. And that will give me one equation for B and C. And then I'm going to do the same thing with another term. The x^2 term on the left-hand side, the coefficient is 1. It's 1 times x^2. On the other side, it's A. Remember I actually have A. So I'm going to put it in, it's 1/2. So this is the A term. And so I get 1/2 x^2. And then the only other x^2 is when this Bx multiplies this x. So Bx * x is Bx^2, so this is-- the other coefficient on x^2 is B. And that forces B to be 1/2.

And last of all, I'm going to do the x^0 term. Or, otherwise known as the constant term. And on the left-hand side, the constant term is 0. There is no constant term. On the right-hand side there's a constant term, 1/2 * 1. That's 1/2 here. And then there's another constant term, which is this constant times this -1 Which is -C. And so the conclusion here is that C = 1/2. Another question. Yeah.

STUDENT: [INAUDIBLE]

PROFESSOR: There's also an x^0 hidden in here. Sorry, an x^1, that's what you were asking about, sorry. There's also an x^1. The only reason why I didn't go to the x^1 is that it turns out with these two I didn't need it. The other thing is that by experience, I know that the extreme ends of the multiplication are the easiest ends. And the middle terms have tons of cross terms. And so I don't like the middle term as much because it always involves more arithmetic. So I stick to the lowest and the highest terms if I can. So that was really a sneaky thing. I did that without saying anything. Yes.

STUDENT: [INAUDIBLE]

PROFESSOR: Another good question. Could I just set x equals 0? Absolutely. In fact, that's equivalent to picking out the x^0 term. And you could plug in numbers. If you wanted. That's another way of doing this besides doing that. So you can also plug in numbers. Can plug in numbers. x = 0. Actually, not x = 1, right? -1, 2, etc. Not 1 just because we've already used it. We won't get interesting information out. Yes.

STUDENT: [INAUDIBLE]

PROFESSOR: So the question is, could I have done it this other way. With the polynomial, with this other one. Yes, absolutely. So in other words what I've taught you now is two choices which are equally reasonable. The one that I picked was the one that was the fastest for this guy and the one that was fastest for this one, but I could've done the other way around. There are a lot of ways of solving simultaneous equations. Yeah, another question.

STUDENT: [INAUDIBLE]

PROFESSOR: The question is the following. So now everybody can understand the question. If this, instead of being $x^2 + 1$, this were $x^3 + 1$. So that's an important case to understand. That's a case in which this denominator is not fully factored. So it's an $x^3 + 1$, you would have to factor out an $x + 1$. So that would be a situation like this, you have an $x^3 + 1$, but that's $(x+1)(x^2 + x + 1)$, this kind of thing. If that's the right, there must be a minus sign in here maybe. OK, something like this. Right? Isn't that what it is?

STUDENT: [INAUDIBLE]

PROFESSOR: I think it's right. But anyway, the point is that you have to factor it. And then you have a linear and a quadratic. So you're always going to be faced eventually with linear factors and quadratic factors. If you have a cubic, that means you haven't factored sufficiently. So you're still back in Step 1.

STUDENT: [INAUDIBLE]

PROFESSOR: In the $x^3 + 1$ case?

STUDENT: [INAUDIBLE]

PROFESSOR: In the $x^3 + 1$ case, we are out of luck until we've completed the factorization. Once we've completed the factorization, we're going to have to deal with these two factors as denominators. So it'll be this plus something over $x + 1$ plus a $Bx + C$ type of thing over this thing here. That's what's eventually going to happen. But hold on to that idea. Let me carry out one more example here.

So I've figured out what all the values are. But I think it's also worth it to remember now that we also have to carry out the integration. What I've just shown you is that the integral of x^2 dx over $(x - 1)(x^2 + 1)$ is equal to, and I've split up into these pieces. So what are the pieces? The pieces are, 1/2, x - 1, plus 1/2 x / $(x^2 + 1)$. This is the A term. This is the B term. And then there's the C term. So we'd better remember that we know how to antidifferentiate these things. In other words, I want to finish the problem. The others were pretty easy, so I didn't bother to finish my sentence, but here I want to be careful and have you realize that there's something a little more to do.

First of all we have the, the first one is no problem. That's this. The second one actually is not too bad either. This is, by the advanced guessing method, my favorite method, something like the logarithm, because that's what's going to appear in the denominator. And then, if you differentiate this, you're going to get 2x over this. But here we have 1/2. So altogether it's 1/4 of this. So I fixed the coefficient here. And then the last one, you have to think back to some level of memorization here and remember that this is 1/2 the arc tangent.

STUDENT: [INAUDIBLE]

PROFESSOR: Why did I go to 1/4? Because in disguise, for this guy, I was thinking d/dx of $\ln(x^2 + 1)$ is 2x / $(x^2 + 1)$. Because it's the derivative of this divided by itself. This is-- The derivative of ln u is u' / u. (ln u)' = u' / u. That was what I applied. And what I had was 1/2, so I need a total of 1/4 to cancel. So 2/4 is 1/2. Now I've got to get you out of one more deep hole. And I'm going to save the general pattern for next time. But I do want to clarify one thing. So let's handle this thing. What if the degree of P is bigger than or equal to the degree of Q. That's the thing that I claimed was easier. And I'm going to describe to you how it's done. Now, in analogy, with numbers you might call this an improper fraction. That's the thing that should echo in your mind when you're thinking about this. And I'm just going to do it by example here. Let's see., I cooked up an example so that I don't make an arithmetic mistake along the way.

So there are two or three steps that I need to explain. So here's an example. The denominator's degree 2, the numerator is degree 3. It well exceeds, so there's a problem here. Our method is not going to work. And the first step that I want to carry out is to reverse Step 1. That is, I don't want the factorization for what I'm going to do next. I want it multiplied out. That means I have to multiply through, so I get $x^2 + x - 2$. I'm back to the starting place here. And now, the next thing that I'm going to do is, I'm going to use long division. That's how you convert an improper fraction to a proper fraction. This is something you were supposed to learn in, I don't know, Grade 4, I know. Grade 3, Grade 4, Grade 5, Grade 6, etc.

So here's how it works in the case of polynomials. It's kind of amusing. So we're dividing this polynomial into that one. And so the quotient to first order here is x. That is, that's going to match the top terms. So I get $x^3 + x^2 - 2x$. That's the product. And now I subtract. And it cancels. So we get here $-x^2 + 2x$. That's the difference. And now I need to divide this next term in. And I need a -1. So -1 times this is $-x^2 - x + 2$. And I subtract. And the x^2's cancel. And here I get $+3x - 2$.

Now, this thing has a name. This is called the quotient. And this thing also has a name. This is called the remainder. And now I'll show you how it works by sticking it into the answer here. The quotient is $x - 1$. And the remainder is, let's get down there. $(3x - 2) / (x^2 + x - 2)$. So the punchline here is that this thing is easy to integrate. This is easy. And this one, you can use, now you can use cover-up, The method that we had before. Because the degree of the numerator is now below the degree of the denominator. It's now first degree and this is second degree. What you can't do is use cover-up to start out with here. That will give you the wrong answer. So that's the end for today, and see you next time.