SYMMETRY RESTRICTIONS FOR 4TH RANK PROPERTY TENSORS

TRICLINIC (21)

$$\begin{matrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ & & S_{33} & S_{34} & S_{35} & S_{36} \\ & & & S_{44} & S_{45} & S_{46} \\ & & & & S_{55} & S_{56} \\ & & & & & S_{66} \end{matrix}$$

MONOCLINIC ($2 \parallel X_3$) (13)

$$\begin{matrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ & S_{22} & S_{23} & 0 & 0 & S_{26} \\ & & S_{33} & 0 & 0 & S_{36} \\ & & & S_{44} & S_{45} & 0 \\ & & & & S_{55} & 0 \\ & & & & & S_{66} \end{matrix}$$

ORTHORHOMBIC ($2 \parallel X_1, X_2, X_3$) (9)

$$\begin{matrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ & S_{22} & S_{23} & 0 & 0 & 0 \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{55} & 0 \\ & & & & & S_{66} \end{matrix}$$

TETRAGONAL ($4, \bar{4}, 4/m$) (7)

$$\begin{matrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ & S_{11} & S_{13} & 0 & 0 & -S_{16} \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{44} & 0 \\ & & & & & S_{66} \end{matrix}$$

TETRAGONAL ($422, 4mm, \bar{4}2m, 4/mmm$) (6)

$$\begin{matrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ & S_{11} & S_{13} & 0 & 0 & 0 \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{44} & 0 \\ & & & & & S_{66} \end{matrix}$$

CUBIC (3)

$$\begin{matrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ & S_{11} & S_{12} & 0 & 0 & 0 \\ & & S_{11} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{44} & 0 \\ & & & & & S_{44} \end{matrix}$$

Symmetry Restrictions for 4th Rank Property Tensors

Triclinic (21)

$$\begin{matrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ & & S_{33} & S_{34} & S_{35} & S_{36} \\ & & & S_{44} & S_{45} & S_{46} \\ & & & & S_{55} & S_{56} \\ & & & & & S_{66} \end{matrix}$$

Monoclinic ($2 \parallel X_3$) (13)

$$\begin{matrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ & S_{22} & S_{23} & 0 & 0 & S_{26} \\ & & S_{33} & 0 & 0 & S_{36} \\ & & & S_{44} & S_{45} & 0 \\ & & & & S_{55} & 0 \\ & & & & & S_{66} \end{matrix}$$

Orthorhombic ($2 \parallel X_1, X_2, X_3$) (9)

$$\begin{matrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ & S_{22} & S_{23} & 0 & 0 & 0 \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{55} & 0 \\ & & & & & S_{66} \end{matrix}$$

Tetragonal ($4, \bar{4}, 4/m$) (7)

$$\begin{matrix} S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\ & S_{11} & S_{13} & 0 & 0 & -S_{16} \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{44} & 0 \\ & & & & & S_{66} \end{matrix}$$

Tetragonal ($422, 4mm, \bar{4}2m, 4/mmm$) (6)

$$\begin{matrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ & S_{11} & S_{13} & 0 & 0 & 0 \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{44} & 0 \\ & & & & & S_{66} \end{matrix}$$

Cubic (3)

$$\begin{matrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ & S_{11} & S_{12} & 0 & 0 & 0 \\ & & S_{11} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{44} & 0 \\ & & & & & S_{44} \end{matrix}$$

HEXAGONAL

TRIGONAL $(3, \bar{3})$

$$\begin{matrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & 0 \\ & S_{11} & S_{13} & -S_{14} & -S_{15} & 0 \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & 0 & -2S_{15} \\ & & & & S_{44} & 2S_{14} \\ & & & & & (2S_{11}-2S_{12}) \end{matrix}$$

(7)

TRIGONAL $(32, 3m, \bar{3}m)$

$$\begin{matrix} S_{11} & S_{12} & S_{13} & S_{14} & 0 & 0 \\ & S_{11} & S_{13} & -S_{14} & 0 & 0 \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{44} & 2S_{14} \\ & & & & & (2S_{11}-2S_{12}) \end{matrix}$$

(6)

HEXAGONAL $(6, \bar{6}, 6/m, 622, 6mm, \bar{6}m2, 6/mmm)$

$$\begin{matrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ & S_{11} & S_{13} & 0 & 0 & 0 \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{44} & 0 \\ & & & & & (2S_{11}-2S_{12}) \end{matrix}$$

(5)

ISOTROPIC

$$\begin{matrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ & S_{11} & S_{12} & 0 & 0 & 0 \\ & & S_{11} & 0 & 0 & 0 \\ & & & 2(S_{11}-S_{12}) & 0 & 0 \\ & & & & 2(S_{11}-S_{12}) & 0 \\ & & & & & 2(S_{11}-S_{12}) \end{matrix}$$

(2)

$$2C_{44} = (C_{11} - C_{12})$$

$$\tfrac{1}{2} S_{44} = (S_{11} - S_{12})$$

CAUCHY RELATION

(1) CENTRAL FORCES

(2) EACH ATOM AT CENTER OF SYMMETRY

(3) NO INITIAL STRESS

$$\boxed{C_{12} = C_{44}}$$