3.60 Symmetry, Structure and Tensor Properties of Materials

Problem Set 12

1. Point out the fallacy, if any, in the following arguments.

   (a) The dipole moment of charges +q and -q separated by distance, d, is a vector quantity defined as $\vec{p} = q\vec{d}$ (the direction of $\vec{d}$ is defined as extending from the negative charge to the positive). Atoms subjected to an electric field, $\vec{E}$, undergo charge deformation which results in an induced dipole moment. The dipole moment is related to the applied field by the ionic electrical polarizability tensor $p_i = \alpha_{ij}E_j$.

   Second-rank tensors are isotropic for cubic crystals.

   Atoms in many cubic crystals are found to have anisotropic polarizability.

   This is a violation of Neumann's Principle.

   (b) The ability of a crystal to diffract x-rays is a property of a crystal. Neumann's Principle states that the symmetry of any property depends on the point group of a crystal, and that the symmetry of a property must be at least that of the point group.

   X-ray diffraction patterns, however, are commonly used to determine the space group of crystals.

   Neumann's Principle therefore does not apply to x-ray diffraction.

2. The Cartesian coordinates of a point, X_i, transform upon a change of coordinate system according to

$$X_i' = c_{ij}X_j$$

   where c_{ij} is the direction cosine scheme which relates the axes of the two coordinate systems. We have stated that if one can, by inspection, write down a relation between the axes of the two coordinate systems x_i and x_j (e.g., $x_1' = -x_1$, $x_2' = -x_2$), that this provides a convenient method for extracting the direction cosine scheme (e.g., $c_{11} = -1$, $c_{12} = 0$, $c_{13} = 0$ etc.).

   The above statement is rather cavalier, however, and not strictly correct: The first equation involves coordinates of a point, while the later equations, written by inspection, involve a vector relationship between reference axes in the two coordinate systems.

   Demonstrate (by means of a sketch or otherwise) that the procedure is correct--that is, that the coordinates of a point transform in exactly the same way as the axes of the reference system.