Solutions Chapter 6

6.1

a) $\alpha_1 = \sin\theta\cos\phi$, $\alpha_1^2 = \sin^2\theta\cos^2\phi$

$\alpha_2 = \sin\theta\sin\phi$, $\alpha_2^2 = \sin^2\theta\sin^2\phi$

$\alpha_3 = \cos\theta$, $\alpha_3^2 = \cos^2\theta$

$\alpha_1^2 + \alpha_2^2 + \alpha_3^2 =$

$\sin^2\theta(\cos^2\phi + \sin^2\phi) + \cos^2\theta = 1$

b) $\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2 = \sin^4\theta\cos^2\phi\sin^2\phi$

$+ \sin^2\theta\sin^2\phi\cos^2\theta + \sin^2\theta\cos^2\phi\cos^2\theta$

$= \sin^4\theta\cos^2\phi\sin^2\phi + \sin^2\theta\cos^2\theta$ QED

6.2 From Eq. 6.6

$f_{100} = K_0$,

$f_{110} = K_0 + K_1/4$, and

$f_{111} = K_0 + K_1/3 + K_2/27$.

For Fe:		From Fig. 6.1	From Eq. 6.6
$f_{111} - f_{100}$	$\approx$	1.6×10^4 J/m^3	$= K_1/3 + K_2/27$
$f_{110} - f_{100}$	$\approx$	1.2×10^4 J/m^3	$= K_1/4$

The second equation gives $K_1 = 4.8 \times 10^4$ J/m^3 and using this in the first gives $K_2 \approx 0$, in fair agreement with the tabulated values, $K_1 = 4.8 \times 10^4$ J/m^3 , $K_2 = -1 \times 10^4$ J/m^3.

For Ni		From Fig. 6.1	From Eq. 6.6
$f_{100} - f_{111}$	$\approx$	2.2×10^3 J/m^3	$= -K_1/3 - K_2/27$
$f_{110} - f_{111}$	$\approx$	1.0×10^3 J/m$_3$	$= -K_1/12 - K_2/27$

Subtracting these two equations gives $K_1 = -4.8 \times 10^3$ J/m^3 and, thus, $K_2 \approx -1.6 \times 10^3$ J/m^3. These values compare well with the tabulated values, $K_1 = -4.5 \times 10^3$ and $K_2 = -2.3 \times 10^3$ J/m^3. Clearly, there is significant opportunity for error in estimating the areas in Fig. 6.1 between the magnetization curves taken in different directions.

6.3 The energy gradient of Eq. 6.6 for small θ is given by $K_1\theta^2 + (K_1 + K_2)\sin^2 2\phi\theta^2/4$. For Ni, both K_1 and K_2 are negative and $K_1 \approx K_2$. Thus the energy gradient is given by $-2|K_1|\ \theta\,[1 + 3/2\ \theta^2 \sin^2 2\ \phi]$ which is steeper for $\phi = 45^o$. Thus M rotates toward the <111> directions, not <110>.

6.4 $f_a^{100} = K_0 + K_1 <sin^4\ \theta\ cos^2\ \phi\ sin^2\ \phi + sin^2\ \theta\ cos^2\ \theta>$

for small θ we get $f_a^{100} \approx K_o + K_1 <\theta>^2$ and

$f_a^{110} = K_0 + K_1\ cos^2\ 2\theta \approx K_0 + K_1 <1 - (2\delta\theta)^2/2 \ldots>^2$

$f_a^{110} - f_a^{100} = K_1 (<1-(2\delta\theta)^2/2 \ldots>^2 - <\delta\theta^2>) = K_1 (1 - 5\ \delta\theta^2>$

and using $m(T) = <1 - \delta\theta^2/2 \ldots>$ for small θ as in text

$\Delta f = (K_1/4)[m]^{10}$

6.5

$f_a^{easy} = K_0 + K_u <sin^2\ \theta> \approx K_0 + K_u <\theta^2>$

$f_a^{hard} = K_0 + K_u <cos^2\ \theta' + cos^2\ \phi'\ sin^2\ \theta'>$

$$\approx K_0 + K_u <(1 - \theta'^2/2)^2 + cos^2\, \phi'\, \theta'^2 >$$

Since $cos^2\, \phi'$ averages to 1/2

$f_a^{hard} = K_0 + K_u <1 - \theta'^2/2>$

and $f_a^{hard} - f_a^{easy} = K_u < 1 - 3\, \delta\theta^2/2... >$ and $m(T) = <cos\, \theta>$

$= <1 - \theta^2/2\, .\, .\, .>$ for θ or θ',

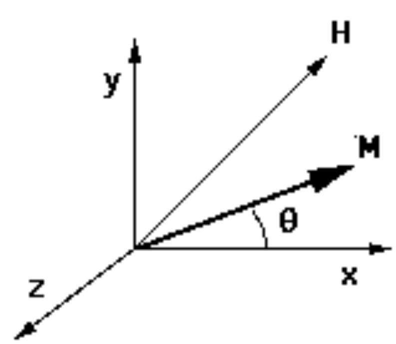
so $f_a^{hard} - f_a^{easy} = K_u\, (T)/K_u\ [0] = [m(T)]^{\,3}$.

6.7 In both cases the question we are asking is what is the measured magnetization in the hard direction after removal of a saturating field that was applied in the hard direction.

For Fe or Ni after magnetization in the hard direction (<111> and <100>, respectively), the magnetization relaxes to the nearest easy axes, distributing itself equally among them: $M_s/3$ along each of the three nearest <100> directions for Fe and $M_s/4$ along each of the four nearest <111> directions for Ni. These axes have projections of $1/\sqrt{3}$ on the original field direction in each case, so the sum over the 3 or 4 near easy axes gives a magnetization component along the hard direction of $M_s/\sqrt{3} = 0.577M_s$, which is observed for both Fe and Ni after magnetization in the hard direction.

Cobalt on the other hand has uniaxial symmetry and after magnetization in the hard base-plane direction, the remanence is zero because the nearest easy axis is the *c* axis, 90 degrees from the base plane which has zero projection in the hard direction.

6.8 In the fully demagnetized state the magnetization is uniformly distributed over the six directions, $\pm x$, $\pm y$, $\pm z$. Application then removal of a field along [110], assuming easy wall motion, will result in a distribution along $+x$ and $+y$ in $H = 0$. So we just use one angular



variable, taken as θ in the figure. To write the energy density, note that

$$\boldsymbol{H}_{110} = \frac{H_0}{\sqrt{2}}(1,1,0) \text{ and } M = M_s(\cos\theta, \sin\theta, 0) \text{ so that}$$

$$-\mu_o \boldsymbol{M}\cdot\boldsymbol{H} = \frac{-\mu_o M_s H}{\sqrt{2}}(\cos\theta + \sin\theta).$$

The normalized component of $\mu_o M$ parallel to H is then given by $m = (\cos\theta + \sin\theta)/\sqrt{2}$, which gives $m = 1$ at saturation, $\theta = 45^o$. So the magnetic energy density is

$$f = -\frac{\mu_o M_s H}{\sqrt{2}}(\cos\theta + \sin\theta) + \frac{K}{4}\sin^2 2\theta$$

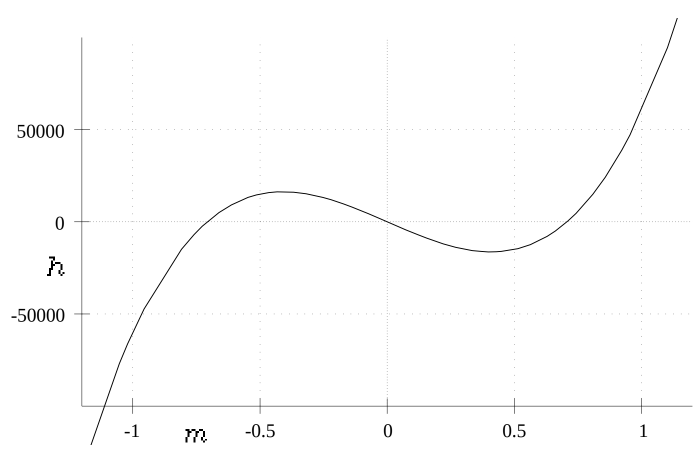
and

$$\frac{\partial f}{\partial\theta} = 0 = \frac{-\mu_o M_s H}{\sqrt{2}}(-\sin\theta + \cos\theta) + K_1 \sin 2\theta \cos 2\theta .$$

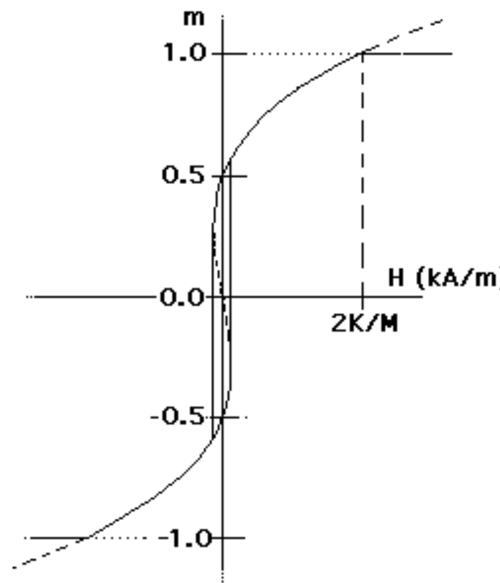
But $\cos 2\theta = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$ so we can cancel the first factor here from the torque equation; it is only zero at and above saturation. Thus, $\mu_o M_s H = \sqrt{2}K_1 \sin 2\theta (\cos\theta + \sin\theta)$. Using $m = (\cos\theta + \sin\theta)/\sqrt{2}$ or $(2m^2-1) = \sin(2\theta)$, the equation of motion is

$$\mu_o M_s H = 2K_1 (2m^2-1)\, m.$$

This can be solved by plotting H vs. m as shown below. Here the values $\mu_o M_s = 2\text{T}$ and $K_1 = 6 \times 10^4\ \text{J/m}^3$ have been used.



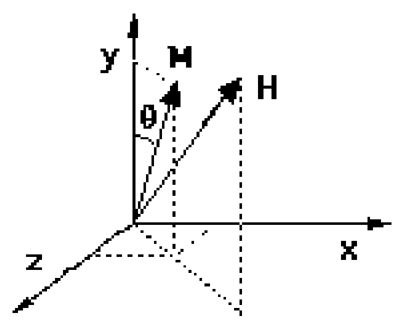
This figure may be plotted as m vs H as shown below. From the analytic solution, it is clear that saturation (m = 1) occurs for $H = 2K_1/\mu_o M_s = H_a$ = 60 kA/m.



The same equation of motion applies to the $\boldsymbol{y}$ component of magnetization. The initial magnetization curve of the component, $M_s/3$, along ± z will involve rotation of that component into + x and + y by 90° wall motion. Thereafter, all of the magnetization proceeds by the derived equation for M_x and M_y.

If wall motion is not easy, one would have to minimize the free energy including the full anisotropy in θ and ϕ.

The case for the field applied along [111] is now treated. $\boldsymbol{H} = H_o(1, 1, 1,)/\sqrt{3}$ and the magnetization process is the same for each Cartesian component of $\boldsymbol{M}$. We treat the component of $\boldsymbol{M}$ that initially lies along z. At arbitrary field it is given by $M(H) = M_s\,(\sin\theta/\sqrt{2},\ \sin\theta/\sqrt{2},\ \cos\theta)$. The Zeeman energy is



$$-\mu_o\, \boldsymbol{M}\cdot\boldsymbol{H} = -(\mu_o M_s H)\,(\sqrt{2}\sin\theta + \cos\theta)/\sqrt{3}.$$

The cubic anisotropy for ϕ = 45° is given by

$$f_a = K_1\left(\frac{\sin^4\theta}{4} + \sin^2\theta\cos^2\theta\right)$$

which has absolute minima at θ = 0 and π as well as at $\theta = \pi/2$ with ϕ = 0, ±π/2 and π. Saddle points can also be identified from Fig. 6.6a).

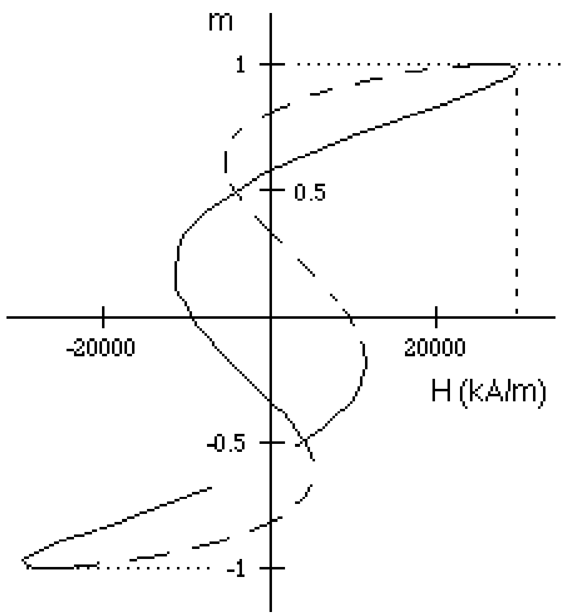
The zero-torque condition is given by:

$$\partial f/\partial\theta = 0 = -\,(\mu_o M_s H_o/\sqrt{3})\,(\sqrt{2}\cos\theta - \sin\theta) + K_1 \sin 2\theta\,(1 + 3\cos 2\theta)/4,$$

which gives the equation of motion

$$H = \frac{\sqrt{3}K_1}{4\mu_o M_s}\frac{\sin(2\theta)[1+3\cos(2\theta)]}{\sqrt{2}\cos\theta - \sin\theta}.$$

This equation can be plotted parametrically with $m = (\sqrt{2}\sin\theta + \cos\theta)/\sqrt{3}$ to give the result shown below. Alternatively, it can be solved analytically (with little further insight) as shown in Cullity, p. 227. The zero-torque solution shown as dashed lines below can be excluded by looking at the stability condition, $d^2f/d\theta^2 > 0$, which is negative for the dashed solutions.



Note that the approach to saturation accelerates as $m \rightarrow 1$. The remanence (at H = 0 or θ = 0) is, from the definition of m, given by $1/\sqrt{3} = 0.577$. As H decreases from positive saturation, the magnetization reaches the extremum in the second quadrant. At this point, it is energetically favorable to jump to the third quadrant solution - if domain wall motion has not already taken the system to that branch.

6.9 The energy surface is described by $E = +2\pi M_s^2 \cos^2\theta - K_u \cos^2\theta$ where θ is the angle between ***M*** and the surface normal. Energy minimization gives $(K_u - 2\pi M_s^2) \sin\theta \cos\theta = 0$ which has solutions at $\theta = 0$ and $\pi/2$ or at $K_u = 2\pi M_s$. Consideration of the stability condition $(K_u - 2\pi M_s^2) \cos 2\theta > 0$ indicates that $\theta = 0$ is the stable condition for $K_u > 2\pi M_s^2$ and $\theta = \pi/2$ for $K_u < 2\pi M_s^2$. Only if K_u is *exactly* equal to $2\pi M_s^2$ could any intermediate orientation exist. There are other forms of anisotropy for which $0 < \theta < \pi/2$ is stable for a range of values of K and M_s.