3.23 Electrical, Optical, and Magnetic Properties of Materials
Fall 2007

Quantum Mechanics - exercice sheet 1

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September 12, 2007

1

In a monoatomic gas, one measure of the "average speed" of a gas particle is the **root mean square speed** defined as follow: $v_{\text{rms}} = \langle v^2 \rangle^{1/2} = \sqrt{\frac{3k_B T}{m}}$, where k_B is the Boltzmann constant, T the temperature, and m the mass of a particle. Using this formula, calculate the **De Broglie** wavelength for Helium (He) and Argon (Ar) atoms at 100K and 500K.
***
Datas:
Helium molar mass, 4.033 g/mole
Argon molar mass, 39.95 g/mole
***

2

Electrons have been used to determine molecular and solid structures using diffraction. Calculate the speed of an electron for which the **De Broglie** wavelength is equal to a typical bond length , namely, 0.150 nm.
***
Datas:
electron mass, $9.109 * 10^{-31}$ kg
***

3

Why can we conclude that the wave function $\psi(x,t) = \phi(x)e^{-\frac{iEt}{\hbar}}$ represents a standing wave?

4

If $\psi(x,t) = A\sin(kx - \omega t)$ describes a wave travelling in the +x direction, how would you describe a wave travelling in the -x direction?

5

Distinguish between the following terms applied to the following set of functions, $\psi_1(x), \psi_2(x), ..., \psi_n(x)$: orthogonal, normalized and complete. Give a mathematical expression to express those terms using integrals.

6

Determine in each of the following cases if the function in the first column of table 1 is an **eigenfunction** of the **opertor** in the second column. If so, what is the corresponding eigenvalue?

wavefunctions	operators
$\sin(\phi)\cos(\phi)$	$\frac{\partial}{\partial\phi}$
$e^{-x^2/3}$	$(\frac{1}{x})\frac{d}{dx}$
xy	$x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}$
$3\cos(\theta)^2 - 1$	$\frac{1}{\sin(\theta)}\frac{d}{d\theta}(\sin(\theta)\frac{d}{d\theta})$
x^2	$\frac{d}{dx}$

Table 1: table of wavefunctions and operators

7

Which of the following wavefunctions are eigenfunctions of the operator $\frac{d}{dx}$? If they are eigenfunctions, what is the associated eigenvalue?

- $ae^{-3x} + be^{-3ix}$
- $\sin^2(x)$
- e^{-ix}
- $\cos(ax)$
- e^{-ix^2}