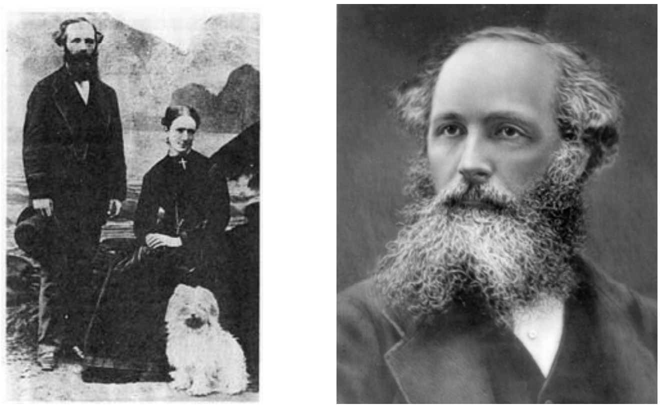
3.23 Electrical, Optical, and Magnetic Properties of Materials
Fall 2007

3.23 Fall 2007 – Lecture 16

MAXWELL AND ELECTROMAGNETISM



James Clerk Maxwell

Cavendish Laboratory

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Last time

1. p-n junctions, built-in voltage, rectification
2. Bloch oscillations, conductivity in semiconductors
3. Electron transport at the nanoscale
4. Phonons, vibrational free energy, and the quasi-harmonic approximation
5. Electron-phonon interactions, and phonon-phonon decays

Study

- Fox, Optical Properties of Solids, Appendix A and Chap 1.
- Prof Fink lecture notes (to be posted)

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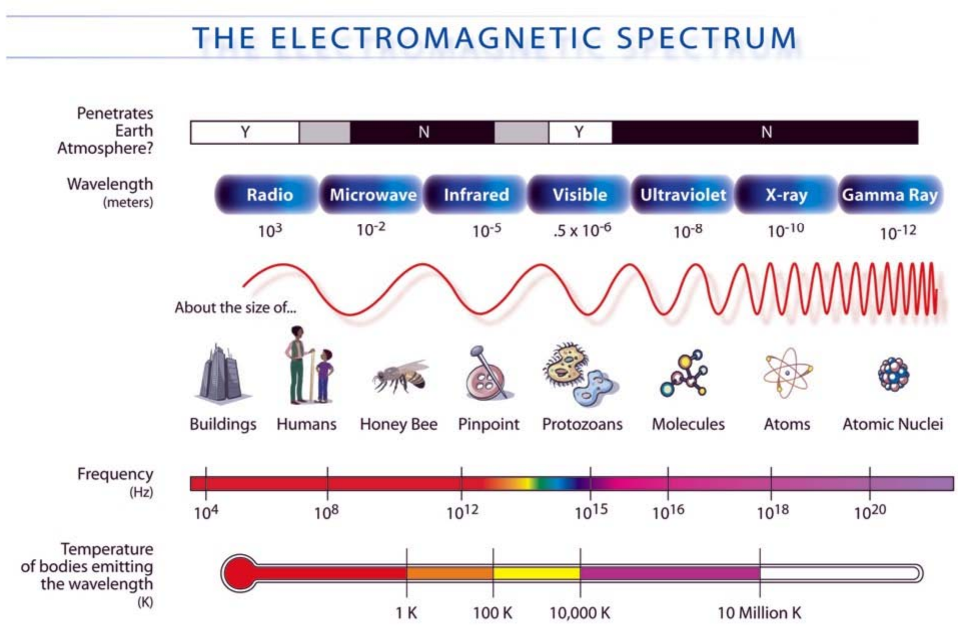

Image courtesy NASA.

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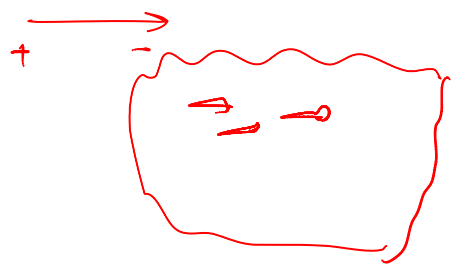
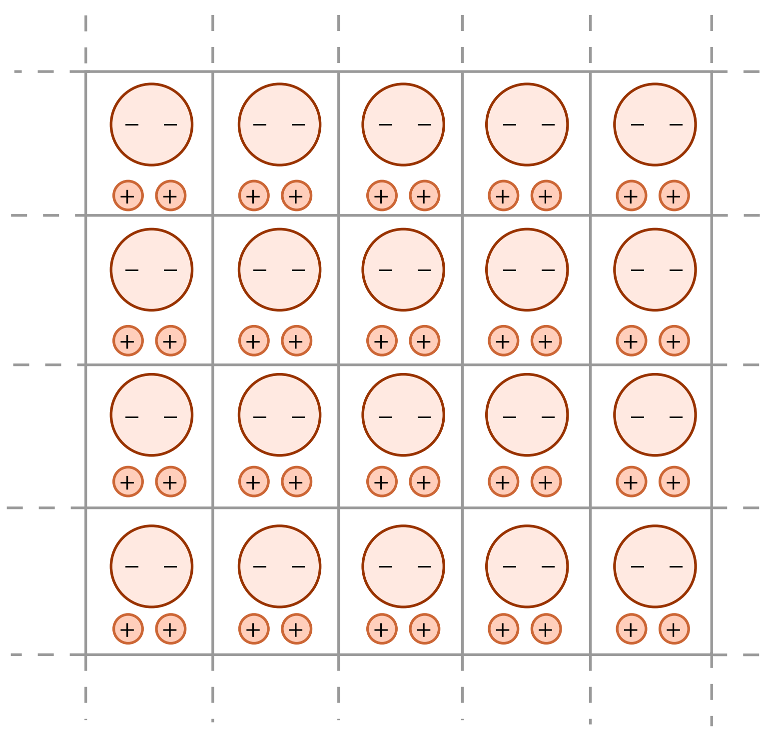
Electric field, polarization, displacement

$\vec{E}$



$\vec{P}$

$\vec{D}$



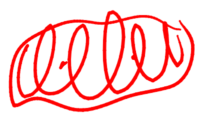
A complex crystal lattice can have a permanent intrinsic polarization P.

Figure by MIT OpenCourseWare.

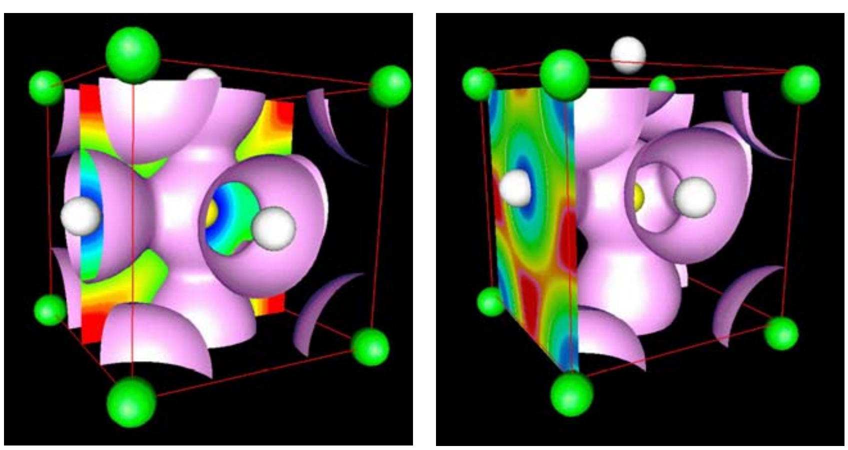
Lines & Glass, *Principles and Applications of Ferroelectrics and Related Materials (1977)*:

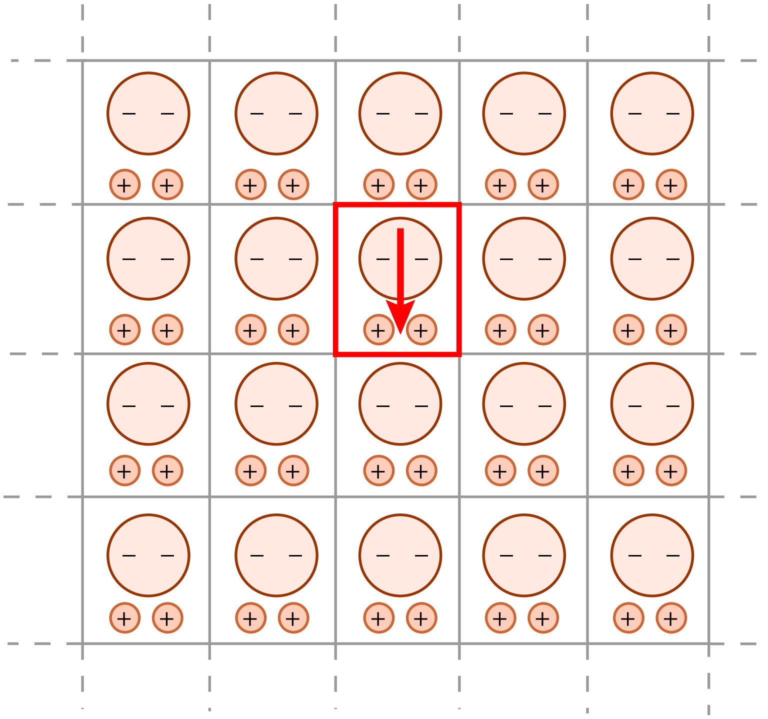
If and when good **electron-density maps** become available for ferroelectrics, expressing charge density $\rho(\mathbf{r})$ as a function of position vector **r** throughout the unit cell, more quantitative estimates of spontaneous polarization might be envisaged as

$$\mathbf{P}_s = \frac{1}{V}\int_V \mathbf{r}\rho(\mathbf{r})\, d\mathbf{r}. \qquad (6.1.19)$$

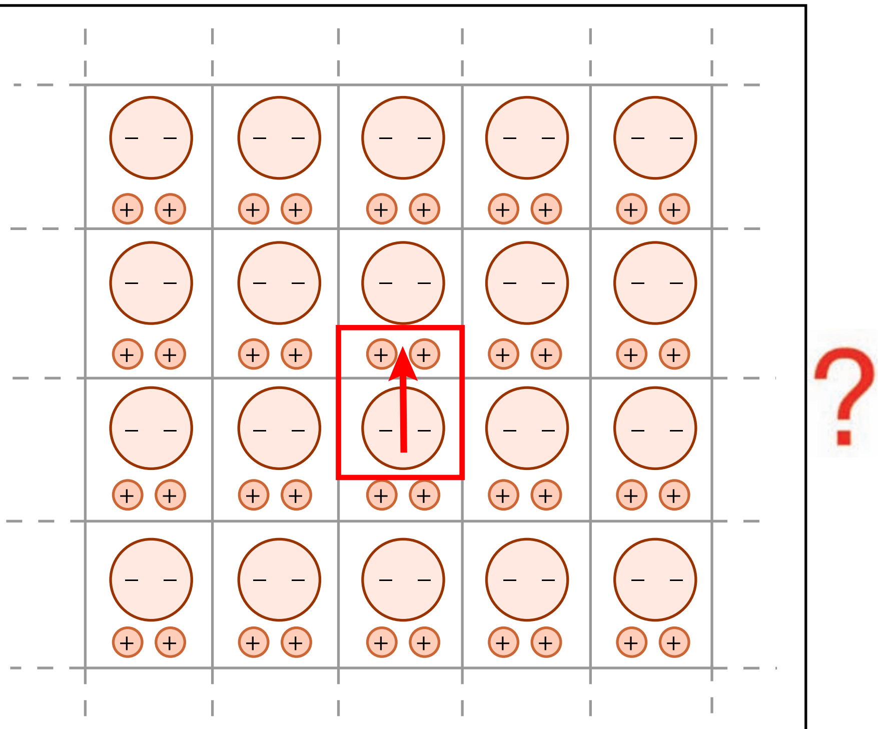


Polarization in lead titanate



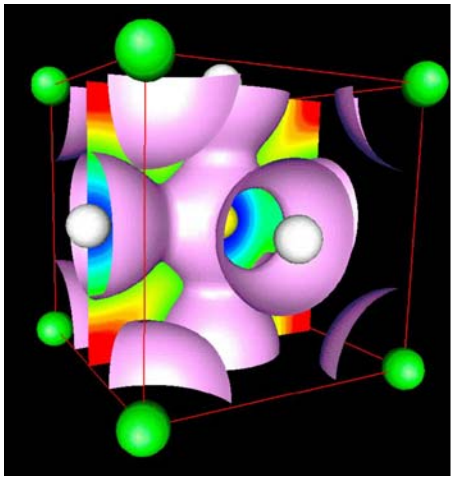


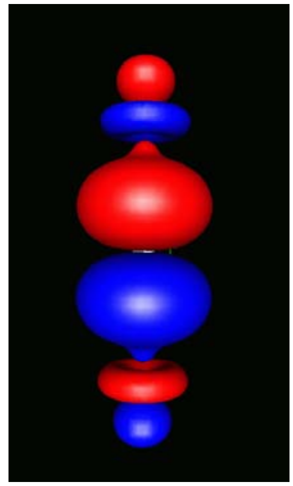
A complex crystal lattice can have a permanent intrinsic polarization P.



A complex crystal lattice can have a permanent intrinsic polarization P.

Figures by MIT OpenCourseWare.





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Dielectric constant, susceptibility

$$\vec{P} = \chi \vec{E} + \left(\chi^{(2)} E^2 + \chi^{(3)} E^3 \ldots \right)$$

↳ SUSCEPTIBILITY

$$\vec{D} = \vec{E} + 4\pi \vec{P} = \vec{E} + 4\pi \chi \vec{E} = \underbrace{(1 + 4\pi\chi)}_{\varepsilon} \vec{E}$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi \rho \qquad \vec{E} = -\vec{\nabla} V$$

STATIC ε_0 DIELECTRIC CONSTANT

(CLAMPED ION) ε_∞ " "

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Magnetic response

$\vec{H}$ = MAGNETIC FIELD

$\vec{M}$ = MAGNETIZATION = $\chi_M \vec{H}$

$\vec{B} = \vec{H} + 4\pi \vec{M} = \vec{H} \underbrace{(1 + 4\pi\chi_M)}_{\text{MAGNETIC PERMEABILITY}}$

MAGNETIC FLUX DENSITY

$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}$

Maxwell equations

$$\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$$

⇒ FARADAY/LENZ

$$\vec{\nabla} \times \vec{H} - \frac{1}{c}\frac{\partial \vec{D}}{\partial t} = \frac{4\pi}{c}\vec{J}$$

⇒ AMPERE'S

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{D} = 4\pi\rho$$

Vector potential and gauges

$$\vec{\nabla}\cdot\vec{B} = 0 \implies \vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{\nabla}\times\left(\vec{\nabla}\varphi\right) = 0 \qquad \vec{A} \mapsto \vec{A} + \vec{\nabla}\varphi$$

$$\boxed{\vec{\nabla}\cdot\vec{A} = 0} \quad \text{COULOMB GAUGE}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\left(\vec{\nabla}\times\vec{A}\right)} = -\frac{1}{c}\vec{\nabla}\times\left(-\frac{\partial A}{\partial t}\right)$$

Vector potential and gauges

$$\vec{E} = -\frac{\partial\vec{A}}{\partial t} + \underbrace{\text{CONST}}_{\text{CURL MUST BE ZERO}}$$

$$= -\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}V$$

Summary

$$\vec\nabla\times\vec E+\frac{1}{c}\frac{\partial\vec B}{\partial t}=0$$

$$\vec\nabla\times\vec H-\frac{1}{c}\frac{\partial\vec D}{\partial t}=\frac{4\pi}{c}\vec J$$

$$\vec\nabla\cdot\vec B=0$$

$$\vec\nabla\cdot\vec D=4\pi\rho$$

$$\vec D=\underbrace{\varepsilon}_{\text{dielectric tensor}}\vec E$$

$$\vec B=\underbrace{\mu}_{\text{permeability tensor}}\vec H$$

$$\vec D=\varepsilon\vec E=\vec E+4\pi\vec P$$

$$\vec B=\mu\vec H=\vec H+4\pi\vec M$$

E – electric field

H – magnetic field

D – electric displacement

B – magnetic displacement

12 variables

8 scalar Maxwell equations

Electromagnetic waves

$$\vec\nabla\times\vec E+\frac{1}{c}\frac{\partial\vec B}{\partial t}=0 \Rightarrow \frac{1}{\mu}\vec\nabla\times\vec E+\frac{1}{c}\frac{\partial\vec H}{\partial t}=0$$

$$\vec\nabla\times\left(\frac{1}{\mu}\vec\nabla\times\vec E\right)+\frac{1}{c}\frac{\partial}{\partial t}\vec\nabla\times\vec H=0 \;;$$

$$\vec\nabla\times\vec H-\frac{1}{c}\frac{\partial\vec D}{\partial t}=0 \qquad \frac{1}{c}\frac{\partial}{\partial t}\vec\nabla\times\vec H=\frac{1}{c}\varepsilon\frac{\partial^2 E}{\partial t^2}$$

$\rho=0$

$\vec J=0$

Electromagnetic waves

$$\vec{\nabla} \times \left(\frac{1}{\mu}\vec{\nabla}\times\vec{E}\right) + \frac{\varepsilon}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right) = \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right) - \nabla^2\vec{A}$$

$$\boxed{\nabla^2\vec{E} - \frac{\mu\varepsilon}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}} = 0 \Rightarrow \omega = \frac{c}{\sqrt{\mu\varepsilon}}|\vec{k}|$$

$$\mu \simeq 1$$

$$\vec{E}(x,y,z,t) = \vec{E}_0 e^{i(\omega t - \vec{k}\cdot\vec{r})}$$

Summary

$$\vec{\nabla}\times\vec{E} + \frac{1}{c}\frac{\partial\vec{B}}{\partial t} = 0 \xrightarrow{\cdot\frac{1}{\mu}} \frac{1}{\mu}\vec{\nabla}\times\vec{E} + \frac{1}{c}\frac{\partial\vec{H}}{\partial t} = 0 \xrightarrow{\cdot\vec{\nabla}\times} \vec{\nabla}\times\left(\frac{1}{\mu}\vec{\nabla}\times\vec{E}\right) + \frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\times\vec{H} = 0$$

$$\vec{\nabla}\times\vec{H} - \frac{1}{c}\frac{\partial\vec{D}}{\partial t} = 0 \xrightarrow{\cdot\frac{\partial}{\partial t}} \frac{1}{c}\frac{\partial}{\partial t}\vec{\nabla}\times\vec{H} = \frac{\varepsilon}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\vec{\nabla}\times\left(\frac{1}{\mu}\vec{\nabla}\times\vec{E}\right) + \frac{\varepsilon}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

$$\vec{\nabla}^2\vec{E} - \frac{\mu\varepsilon}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

$$\vec{\nabla}^2\vec{H} - \frac{\mu\varepsilon}{c^2}\frac{\partial^2\vec{H}}{\partial t^2} = 0$$

$$\vec{\nabla}^2\vec{E} = \frac{\mu\varepsilon}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\vec{E}(x,y,z,t) = \vec{E}_0 e^{i\omega t - \vec{k}\cdot\vec{r}}$$

$$\frac{c}{\sqrt{\mu\varepsilon}}|\vec{k}| = \omega$$

Refractive index

FROM VACUUM TO CONDENSED

$$c \to c/n$$

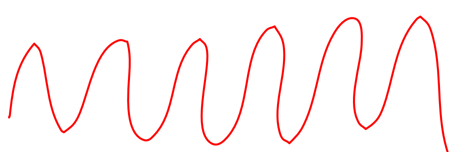
$$\boxed{k = \frac{2\pi}{\lambda} = \frac{\omega}{c}} \Rightarrow \boxed{k = \frac{\omega n}{c}}$$

$$\varepsilon \omega^2 = c^2 k^2 = n^2 \omega^2$$

$$n = \sqrt{\varepsilon}$$

Phase velocity

$$\vec{E}(\vec{r},t) = \vec{E}_0 \, e^{i(\omega t - \vec{k}\cdot\vec{r})}$$



$$\omega t - \vec{k}\cdot\vec{r} = \text{CONST} =$$

$$= \omega(t+\Delta t) - k(\vec{r}+\Delta\vec{r})$$

$$\omega \Delta t = k \Delta r \qquad v = \frac{\omega}{k}$$

Wave packets

$$E(\vec{r},t) = \int \alpha_\omega \, e^{i(\omega t - \vec{k}\cdot\vec{r})} \, d\omega$$

$\omega - \Delta\omega$ $\quad$ $\omega + \Delta\omega$ $\quad$ ω

$\Delta\omega$

$$v = \frac{\Delta\omega}{\Delta k} = \frac{\partial\omega}{\partial k}$$