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3.23 Electrical, Optical, and Magnetic Properties of Materials

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Homework # 7

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1 Intrinsic semiconductor

Let us consider a 2 band intrinsic semiconductor. From the microscopic point of view this semiconductor is characterized by a **valence density of states**, $D_v(E)$, a **conduction density of states**, $D_c(E)$, and a band gap $E_g = E_c - E_v$. Using those density of states and the Fermi factor $f(T, E) = \frac{1}{1+e^{(E-\mu)/k_B T}}$, we can easily express the volumic density of electrons in the conduction band, $n_c(T, \mu)$, and the volumic density of holes in the valence band, $p_v(T, \mu)$:

$$n_c(T, \mu) = \int_{E_c}^{+\infty} f(T, E) D_c(E) dE \text{ and } p_v(T, \mu) = \int_{-\infty}^{E_v} (1 - f(T, E)) D_v(E) dE$$

Those formulas are extremely general and always applicable in practice. Now we will use the hypothesis of **non-degeneracy**, meaning that the chemical potential is far away in terms of thermal energy from the top of the valence band **and** the bottom of the conduction band:

$$\mu - E_v \gg k_B T \text{ and } E_c - \mu \gg k_B T \text{ non-degeneracy conditions}$$

For **intrinsic** semiconductors like Silicon, Germanium and Galium Arsenide those conditions of non-degeneracy are very well satisfied. Using this hypothesis, we see that the Fermi factor can be simplified to a Maxwell-Boltzmann factor (here in the case where $E > \mu$):

$$f(T, E) = \frac{1}{1+e^{(E-\mu)/k_B T}} \text{ becomes } f(T, E) \approx e^{-(E-\mu)/k_B T}$$

1.1 Some general results for any $D_c(E)$ and $D_v(E)$ and both intrinsic and extrinsic semiconductors

1) Using the non-degeneracy conditions, express the densities of electrons and holes, $n_c(T, \mu)$ and $p_v(T, \mu)$, as follows:

- $n_c(T, \mu) = N_c(T) e^{-(E_c-\mu)/k_B T}$
- $p_v(T, \mu) = P_v(T) e^{-(\mu-E_v)/k_B T}$

Find an integral expression for both $N_c(T)$ and $P_v(T)$. What happens when one multiplies $n_c(T)$ by $p_v(T)$?

Important Remark: This relation is called the **law of mass action**. It is satisfied for both intrinsic and extrinsic semiconductors. All the doping does is to introduce some states inside the band gap, and shift the chemical potential, but this does not change the densities of states $D_v(E)$ and $D_c(E)$ so it does not change the law of mass action.

solution:

In the case of electrons, we see that the energy E is always greater than the bottom of the conduction band. So if we use the non-degeneracy condition for the conduction band, we have $E - \mu > E_c - \mu \gg k_B T$, and we can neglect the 1 in front of the exponential $e^{\beta(E-\mu)}$ safely such that we end up with:

$$n_c(T, \mu) \approx \int_{E_c}^{+\infty} D_c(E) e^{-\beta(E-\mu)} dE$$

Now in the exponential we write $E - \mu = E - E_c + E_c - \mu$ and we find:

$$n_c(T, \mu) \approx \left(\int_{E_c}^{+\infty} D_c(E) e^{-\beta(E-E_c)} dE \right) e^{-\beta(E_c-\mu)}$$

In the case of holes, we have a completely symmetrical situation. We know that:

$$p_v(T, \mu) = \int_{-\infty}^{E_v} (1 - f(T, E)) D_v(E) dE = \int_{-\infty}^{E_v} D_v(E) \frac{1}{1 + e^{\beta(\mu-E)}} dE$$

But if we have $\mu - E > \mu - E_v \gg k_B T$, then we can safely neglect the 1 in front of the exponential, and we end up with:

$$p_v(T, \mu) \approx \left(\int_{-\infty}^{E_v} D_v(E) e^{-\beta(E_v-E)} dE \right) e^{-\beta(\mu-E_v)}$$

The expressions for $N_c(T)$ and $P_v(T)$ are then:

$$N_c(T) = \int_{E_c}^{+\infty} D_c(E) e^{-\beta(E-E_c)} dE \text{ and } P_v(T) = \int_{-\infty}^{E_v} D_v(E) e^{-\beta(E_v-E)} dE$$

When one multiplies $n_c(T, \mu)$ by $p_v(T, \mu)$, one realizes that the product is **independent of the chemical potential!**

2) Now if we consider an **intrinsic semiconductor**, what is the relationship between $n_c(T, \mu)$ and $p_v(T, \mu)$? Use this relationship to express the chemical potential as a function of T only. What is the chemical potential at $0K$? What is the order of magnitude of the chemical potential shift at finite T from its value at $T = 0K$? In which direction does the chemical potential shift from its value at $T = 0K$ when one increases the temperature?

solution:

In the case of an **intrinsic** semiconductor absolutely no impurities are present. Therefore when one electron is thermally excited through the band gap, exactly one hole is created in the valence band. We conclude that at all temperatures the total number of conduction electrons is strictly equal to the total number of valence holes.

For **intrinsic semiconductors**, $p_v(T, \mu) = n_c(T, \mu)$

Using this relation plus the hypothesis of non-degeneracy we find:

$$\mu(T) = \frac{E_c + E_v}{2} + \frac{1}{2} k_B T \ln \left(\frac{P_v(T)}{N_c(T)} \right)$$

We see that the chemical potential goes to the middle of the band gap when the temperature goes to zero:

$$\mu(T = 0K) = \frac{E_c + E_v}{2} \text{ for intrinsic semiconductor}$$

The typical value for the chemical potential shift at finite temperature is of order $k_B T$. On this formula, we can also conclude that the chemical potential **moves away** from the band with **the highest effective density of states**. For example if the effective density of states in the valence band is greater than the one in the conduction band, $\frac{P_v(T)}{N_c(T)} > 1$, then the logarithm is positive and the chemical potential moves up in energy, i.e it moves towards the bottom of the conduction band.

1.2 Introducing the parabolic approximation for the valence and conduction bands

3) By using the following results from the parabolic approximation of the valence and conduction bands:

$$D_c(E) = \frac{1}{2\pi^2} \left(\frac{2m_c^*}{\hbar^2}\right)^{3/2} \sqrt{E - E_c} \text{ and } D_v(E) = \frac{1}{2\pi^2} \left(\frac{2m_v^*}{\hbar^2}\right)^{3/2} \sqrt{E_v - E}$$

calculate explicitly $N_c(T)$ and $N_v(T)$. Then use this to calculate the chemical potential μ as a function of T , m_c^* , m_v^* , E_v and E_c . By taking the origin of the energy axis in the middle of the band gap, calculate the chemical potential shift in eV at 300K for GaAs ($m_c^* = 0.063 m$ and $m_v^* = 0.505 m$, where m is the mass of a free electron).

solution:

From the formula found in question 1) plus the analytical form for the densities of states, we find:

$$N_c(T) = \int_{E_c}^{+\infty} D_c(E) e^{-\beta(E - E_c)} dE = \frac{1}{2\pi^2} \left(\frac{2m_c^* k_B T}{\hbar^2}\right)^{3/2} \int_0^{+\infty} \sqrt{x} e^{-x} dx$$

This integral is a classical gaussian integral (or you can use Mathematica to find it), and its value is: $\frac{\sqrt{\pi}}{2}$, such that the final answer is:

$$N_c(T) = \frac{\sqrt{\pi}}{4\pi^2} \left(\frac{2m_c^* k_B T}{\hbar^2}\right)^{3/2}$$

Similarly we find for the holes:

$$P_v(T) = \frac{\sqrt{\pi}}{4\pi^2} \left(\frac{2m_v^* k_B T}{\hbar^2}\right)^{3/2}$$

By plugging-in the values for $P_v(T)$ and $N_c(T)$ found above inside the general formula for the chemical potential of an intrinsic semiconductor, we easily find that:

$$\mu(T) = \frac{E_c + E_v}{2} + \frac{1}{2} k_B T \ln\left(\frac{P_v(T)}{N_c(T)}\right) = \frac{E_c + E_v}{2} + \frac{3}{4} k_B T \ln\left(\frac{m_v^*}{m_c^*}\right)$$

Using the above formula it is easy to calculate the shift in chemical potential of GaAs from the zero temperature value to room temperature:

$$\Delta\mu(0 \rightarrow 300) = \frac{3}{4} k_B \ln\left(\frac{m_v^*}{m_c^*}\right) \Delta T \approx 0.040 \text{ eV}$$

This number should be compared to the band gap of GaAs at 300K which is 1.43 eV. Exactly like in the case of metals, the shift in chemical potential with temperature for a pure semiconductor is very small.

2 p-doped semiconductors

We will now consider a semiconductor homogeneously doped with acceptors. Those acceptors are what we call "shallow impurities" in the sense that their presence inside the host semiconductor is responsible for the appearance of allowed energy states for holes right above the top of the valence band. The concentration of acceptors is N_a and the binding energy of the holes is $\epsilon_a = E_a - E_v$.

2.1 qualitative description of the physics with increasing temperature

4) Describe qualitatively what happens to the majority carriers (in this p-doped semiconductor the majority carriers are the holes) as one increases slowly the temperature T from 0K to high temperature. Use the terms : freezing-out regime, saturation regime and intrinsic regime to specify the different behavior regimes of the doped semiconductor with temperature.

solution:

At very low temperature, everything is "frozen". This means that the acceptor levels are all occupied by one hole, there is no hole in the valence band and the conduction band is empty of electrons. This region in temperatures is called the "freeze-out" region.

At intermediate temperatures, T is high enough to excite holes localized around impurities into the valence band. The typical temperature in this region called the "saturation" region is $T_a = \frac{\epsilon_a}{k_B}$.

At high temperature, T is so high that direct excitation of electrons from the valence to the conduction band is possible. When operating in this region, the doped semiconductor behaves as if it would be intrinsic because the density of carriers in the conduction band becomes quickly much more important than the concentration of impurities. This region is the "intrinsic" region.

2.2 quantitative calculation of the majority carrier density with temperature

5) Let us now construct the curve $\ln(p_v(T))$ as a function of $1/k_B T$. We will study the different regimes that you outlined above and find the expression for the chemical potential and the volumic density of holes in the valence band at different temperatures. To do this we will establish a **general balance equation** for the charges in the system. We denote by $P(E_a, T)$ the probability for a hole to occupy an acceptor level at temperature T . Show that we have the following equation between the total density of electrons in the conduction band $n_c(T, \mu)$, the total density of holes in the valence band $p_v(T, \mu)$ and the concentration of acceptor impurities N_a :

$$n_c(T, \mu) + N_a(1 - P(E_a, T)) = p_v(T, \mu) \text{ balance equation}$$

solution:

A semiconductor, may it be doped or not, carries no extra charge. So the total number of negative charges in the system must always balance the total number of positive charges. The only negative charges in a p-doped semiconductor are electrons and ionized impurities. The only positive charges in a p-doped semiconductor are holes. So the total number of holes in the valence band must balance the total number of electrons in the conduction band plus the total number of ionized impurities at a given temperature. Another way to see this is to say that a hole is created in the valence band by only two possible mechanisms: one in which an electron has been directly excited from the valence band to the conduction band, and one in which a hole has left a neutral impurity to go into the valence band. Using this argumentation, we see that:

$$n_c(T, \mu) + N_a(T, \mu) = p_v(T, \mu)$$

where I used the notation $N_a(T, \mu)$ to indicate the number of ionized acceptors at a given temperature. This number is simply the product of the total number of impurities multiplied by the probability that **one** impurity is ionized. But this probability is nothing more than the probability for a hole not to occupy an impurity state. This probability is then just $1 - P(E_a, T)$. In the end the balance equation takes the following form:

$$n_c(T, \mu) + N_a(1 - P(E_a, T)) = p_v(T, \mu)$$

6) **At very low temperature**, which term in the balance equation can we safely neglect and why? The probability $P(E_a, T)$ is given by $P(E_a, T) = \frac{1}{\frac{1}{2}e^{\frac{\mu - E_a}{k_B T}} + 1}$. In the low temperature limit the chemical potential sits between the top of the valence band E_v and the acceptor level E_a . Simplify the term $1 - P(E_a, T)$ to a single exponential term up to a constant in front, and then use this simplified expression to show that the chemical potential can be written as follows (to obtain this expression you need to use the general expression for $p_v(T, \mu)$ found in the first part of the problem set):

$$\mu(T) = \frac{E_a + E_v}{2} - \frac{1}{2}k_B T \ln\left(\frac{N_a}{2P_v(T)}\right)$$

solution:

At very low temperature, because of the Fermi factor, there is no chance whatsoever that an electron will be thermally excited across the band gap. So $n_c(T, \mu)$ may be safely neglected. The balance equation simplifies into the following:

$$N_a \frac{1}{2} \frac{e^{\beta(\mu - E_a)}}{1 + \frac{1}{2}e^{\beta(\mu - E_a)}} = P_v(T) e^{-\beta(\mu - E_v)}$$

Now we know that $E_v < \mu < E_a$, and so as T goes to zero $\beta(\mu - E_a)$ goes to $-\infty$. We can then neglect the exponential in the denominator of the left-hand side. The equation then reduces to:

$$N_a \frac{1}{2} e^{\beta(\mu - E_a)} = P_v(T) e^{-\beta(\mu - E_v)}$$

which gives us right away:

$$\mu(T) \approx \frac{E_v + E_a}{2} - \frac{1}{2}k_B T \ln\left(\frac{N_a}{2P_v(T)}\right)$$

7) Using the expression for the chemical potential in 6), express $\ln(p_v(T, \mu))$ as a function of T only. What is the coefficient of the linear dependance of $\ln(p_v(T, \mu))$ with $1/k_B T$? This regime is called the **freezing-out regime**.

solution:

Using the equation for the chemical potential established in question 6), plus the general expression for $p_v(\mu, T)$, one finds:

$$\ln(p_v(\mu, T)) = \ln(P_v(T)) - \beta(\mu - E_v) = \ln(P_v(T)) - \beta\left(\frac{E_a - E_v}{2} - \frac{1}{2}k_B T \ln\left(\frac{N_a}{2P_v(T)}\right)\right)$$

This equation can be further simplified:

$$\ln(p_v(\mu, T)) = \frac{1}{2} \ln\left(\frac{N_a P_v(T)}{2}\right) - \frac{\epsilon_a}{2} \left(\frac{1}{k_B T}\right)$$

The coefficient of the linear dependance in $1/k_B T$ is $-\frac{\epsilon_a}{2}$.

8) **At intermediate temperatures**, where all the impurities have been ionized but no electrons have been promoted from the valence to the conduction band, simplify the balance equation and find an expression for the chemical potential. Show that in this regime called the **saturation regime**, $\ln(p_v(T, \mu))$ is independant of the temperature. Define a temperature that will characterize this intermediate regime.

solution:

At intermediate temperatures, all impurities have been ionized and so we have $N_a(1 - P(E_a, T)) \approx N_a$. $n_c(\mu, T)$ is again negligible and the balance equation reduces to:

$$N_a = P_v(T)e^{-\beta(\mu - E_v)}$$

From this we right away conclude that:

$$\mu(T) = E_v - k_B T \ln\left(\frac{N_a}{P_v(T)}\right)$$

We also deduce from the above equations that the concentration of holes in the valence band is independant of the temperature. We trivially have $\ln(p_v(\mu, T)) = \ln(N_a)$. The characteristic temperature above which most of the impurities get ionized is $T_a = \frac{\epsilon_a}{k_B}$.

9) **At very high temperature**, electrons get promoted from the valence band to the conduction band. The density of impurities is then negligible. Use the balance equation to calculate the chemical potential. Using this expression, calculate $\ln(p_v(T, \mu))$ as a function of T only. This regime is called the **intrinsic regime**. What is the obvious reason for this? What is the coefficient of the linear dependance of $\ln(p_v(T, \mu))$ with $1/k_B T$?

solution:

At very high temperatures, a lot of electrons get promoted to the conduction band. So quickly the density of holes created in the valence band get dominated by this "intrinsic" behavior, hence the origin of the "intrinsic regime" of a doped semiconductor. If one then neglect N_a with respect to $p_v(\mu, T)$ in the balance equation, one reduces this equation to the exact equation defining a pure semiconductor:

$$n_c(\mu, T) = p_v(\mu, T)$$

And so the chemical potential is given by the equation found in question 2), $\mu(T) = \frac{E_c + E_v}{2} + \frac{1}{2}k_B T \ln\left(\frac{P_v(T)}{N_c(T)}\right)$. Plugging in the value for the chemical potential in the expression for $p_v(\mu, T)$, we find:

$$\ln(p_v(\mu, T)) = \frac{1}{2} \ln(P_v(T)N_c(T)) - \frac{E_c - E_v}{2} \left(\frac{1}{k_B T}\right)$$

The coefficient of the linear dependance in $1/k_B T$ is $-\frac{E_g}{2}$.

10) Draw a schematic diagram of $\ln(p_v(T, \mu))$ as a function of $1/k_B T$.

solution:

Everything that has been said about the different conduction regimes has been summarized on figure 1.

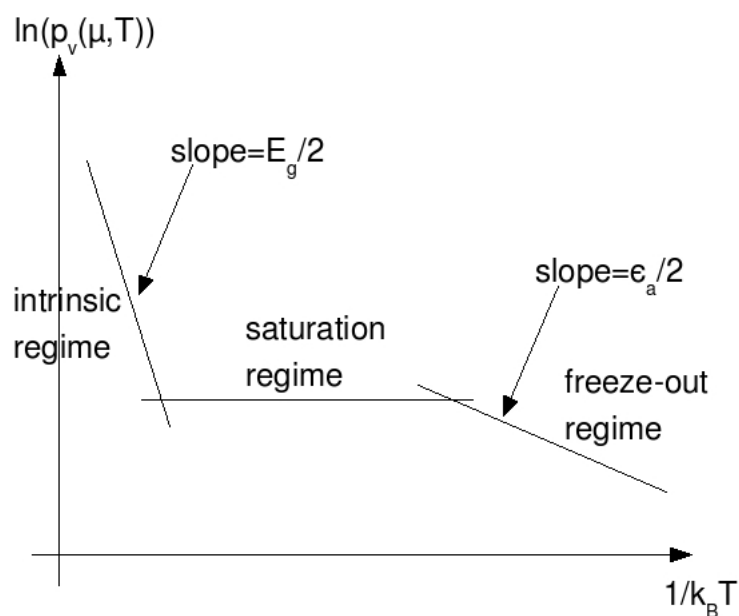


Figure 1: The three different conduction regimes for a doped semiconductor. The low temperature regime (freeze-out regime) shows a straight line of slope $-\frac{\epsilon_a}{2}$. The saturation regime dominated by the impurities is a horizontal line and the high temperature regime (intrinsic regime) shows a straight line of slope $-\frac{E_g}{2}$. This slope is much more important than the freeze-out slope because impurities binding energy are small compared to the semiconductor band gap.

3 The p-n junction

We discussed the physics of what's happening when one joins a p-type semiconductor and an n-type semiconductor together. After a very small amount

of time, a built-in potential is created and a space charge region at the metallurgical junction appears. We consider both doped semiconductor to be homogeneously doped with impurity concentrations N_a and N_d . We choose an x axis perpendicular to the pn junction and going from the p-side to the n-side. At equilibrium, the drift currents and the diffusion currents balance each other. The drift currents are given by the standard Ohm's law:

$$J_n^{drift}(x) = n_c(T, \mu)e\mu_n E \text{ and } J_p^{drift}(x) = p_v(T, \mu)e\mu_p E \text{ drift currents for electrons and holes}$$

In those expressions, μ_n and μ_p are the electron and hole mobilities. The diffusion currents are given by Fick's law:

$$J_n^{diff}(x) = -(-e)D_n \frac{dn_c(T, \mu)}{dx} = eD_n \frac{dn_c(T, \mu)}{dx} \text{ and}$$

$$J_p^{diff}(x) = -(+e)D_p \frac{dp_v(T, \mu)}{dx} = -eD_p \frac{dp_v(T, \mu)}{dx}$$

Remember that electrons have a negative charge $-e$ and the holes have a positive charge $+e$, hence the above equations. Because of the space charge region, an electrical potential $\phi(x)$ is created, and its effect is to shift rigidly the band structure at point x by an amount $-e\phi(x)$ which represents the amount of supplemental potential energy of an electron due to the electric field. We will study the behavior of the pn junction at room temperature which is about 300K.

3.1 calculation of the built-in potential

11) With what has been done in the previous part of the homework, conclude about the behavior regime of the p-side and the n-side at 300K. What is the expression for the chemical potential on the p-side, $\mu_L(T)$, previous to the formation of the junction? Identically one can show that the chemical potential on the n-side previous to the formation of the junction is:

$$\mu_R(T) = E_c + k_B T \ln\left(\frac{N_d}{N_c(T)}\right)$$

where E_c is the bottom of the conduction band, N_d is the concentration of donor impurities on the n-side and $N_c(T)$ is the effective density of states in the conduction band.

solution:

At 300K most of the impurities in both semiconductors are ionized, so each of them is in its saturation regime. The chemical potential in this regime has been calculated in question 8) for the p-doped semiconductor:

$$\mu(T) = E_v + k_B T \ln\left(\frac{P_v(T)}{N_a}\right) \text{ for a p-doped semiconductor in the saturation regime}$$

The exact symmetric result for the n-doped semiconductor is:

$$\mu(T) = E_c + k_B T \ln\left(\frac{N_d}{N_c(T)}\right) \text{ for an n-doped semiconductor in the saturation regime}$$

12) By definition the built-in potential is given by $e\phi_b = E_v(+\infty) - E_v(-\infty) = E_c(+\infty) - E_c(-\infty)$. By observing that the distance between the chemical potential and the top of the valence band must not change on both the p-side and the n-side in the respective limits $x \rightarrow -\infty$ and $+\infty$ even after the junction has been created, prove that the built-in potential is given by:

$$e\phi_b = \mu_R(T) - \mu_L(T)$$

and calculate explicitly this built-in potential as a function of E_g , T , N_a , N_d , $N_c(T)$ and $P_v(T)$. What is the built-in potential at 0K? What is the built-in potential for GaAs at 0K and 300K with uniform doping of $N_a = N_d = 10^{22} \text{ m}^{-3}$? The band gap for GaAs at 300K is 1.43 eV.

solution:

Before we assemble the two semiconductors together to create a junction, the chemical potential and the top of the valence band in the p-doped semiconductor have some definite values: $\mu_L(T)$ and E_v^L . Similarly, in the n-doped semiconductor we have some definite values $\mu_R(T)$ and E_v^R . **After** we join the two semiconductors, the chemical potential has become μ everywhere. Moreover E_v^L and E_v^R have been shifted by the electrical potential by some values $e\phi_L$ and $e\phi_R$. But deep on the p-side and deep on the n-side, the distance between μ and E_v must not have changed because we are far from the junction. So we have:

$$\mu_L(T) - E_v^L = \mu - (E_v^L - e\phi_b) \text{ and } \mu_R(T) - E_v^R = \mu - (E_v^R - e\phi_R)$$

The terms E_v^L and E_v^R disappear on both sides of the above equations and taking the difference between the two above equations leads us to:

$$\mu_R(T) - \mu_L(T) = (\mu + e\phi_R) - (\mu + e\phi_L) = e(\phi_R - \phi_L) = e\phi_b$$

Now using the expressions for the chemical potentials obtained in question 11), we find:

$$e\phi_b = E_c - E_v + k_B T \ln\left(\frac{N_a N_d}{N_c(T) P_v(T)}\right)$$

Remark: here a **little subtlety** appears. Indeed in the above equation E_c is the bottom of the conduction band of the **host material** which is n-doped whereas E_v is the top of the valence band of the **host material** which is p-doped. So **only** in the case where the two host materials are identical, we can say that $E_c - E_v$ **is the band gap**.

At 0K, the built-in potential is just the band gap. For GaAs, we have $e\phi_b(0K) = 1.43 \text{ eV}$ and $e\phi_b(300K) = 1.16 \text{ eV}$. I used the effective masses given earlier to calculate the built-in potential at 300K.

3.2 electrical potential profile and size of the depletion region

Let us now focus on the depletion region. We would like to calculate the size of the depletion region as a function of some fundamental physical constants of the pn junction. For this we will consider that the charge density $\rho(x)$ looks like the following:

$$\rho(x) = -eN_a \text{ for } -d_p < x < 0 \text{ and } \rho(x) = +eN_d \text{ for } 0 < x < d_n$$

where d_p and d_n denote the size of the depletion region on respectively the p-side and the n-side. The way to find the electrical potential is to use the Poisson equation with the appropriate boundary conditions. The Poisson equation in the international system of units is:

$$\frac{d^2\phi(x)}{dx^2} = -\frac{\rho(x)}{\epsilon}$$

where ϵ is the dielectric constant of the medium (here some GaAs with $\epsilon = 13.2 * \epsilon_0$) and ϵ_0 is the vacuum dielectric permittivity $\epsilon_0 = 8.854 * 10^{-12} F.m^{-1}$.

13) Find the boundary conditions that $\phi(x)$ must satisfy and solve the Poisson equation explicitly using the given shape for the charge density $\rho(x)$.

solution:

At minus infinity, the electrical potential must saturate to a given value such that the electric field vanishes. Since potentials are always given up to a constant, let us take $\phi(-\infty) = \phi(x < -d_p) = 0$. At plus infinity the same kind of analysis applies. But since we have a finite potential difference between right and left, we will take $\phi(+\infty) = \phi(x > d_n) = \phi_b$. Moreover because the charge density jumps at $x = -d_p$ and $x = d_n$ are **finite**, the first derivative must be continuous at those points. Since the derivative is zero for both $x < -d_p$ and $x > d_n$, we then have $\frac{d\phi(x)}{dx}|_{x=-d_p} = 0$ and $\frac{d\phi(x)}{dx}|_{x=d_n} = 0$.

The Poisson equation is $\frac{d^2\phi(x)}{dx^2} = -\frac{\rho(x)}{\epsilon}$, and so the solution is very easy to find given the charge density. On the left hand side of the junction we have:

$$\phi(x) = -\frac{eN_a}{2\epsilon}x^2 + \alpha x + \beta \text{ with } -d_p < x < 0$$

and with $\phi(-d_p) = 0$ and $\frac{d\phi(x)}{dx}|_{x=-d_p} = 0$ we can extract α and β and end up with:

$$\phi(x) = \frac{eN_a}{2\epsilon}(x + d_p)^2 \text{ with } -d_p < x < 0$$

In a similar way we can obtain the potential on the right-hand side and find:

$$\phi(x) = \phi_b - \frac{eN_d}{2\epsilon}(x - d_n)^2 \text{ with } 0 < x < d_n$$

14) Using the continuity of the electrical potential and its first derivative (the electric field) at $x = 0$, find two equations that will link the variables N_a , N_d , ϵ , d_p , d_n , e and ϕ_b together. From those expressions, give the values for d_n , d_p and $w = d_p + d_n$ as functions of N_d , N_a , e , ϵ and ϕ_b only. Find the numerical values for each of those length in the case of a GaAs pn junction with $\phi_b = 1V$, $N_a = N_d = 10^{22} m^{-3}$.

solution:

We will use the continuity of the electrical potential and of its first derivative at the interface between the two semiconductors to extract two new relationships. The continuity of the electrical potential gives us:

$$\phi_b = \frac{e}{2\epsilon}(N_a d_p^2 + N_d d_n^2)$$

The continuity of the electric field gives us:

$$\frac{eN_a}{\epsilon} d_p = \frac{eN_d}{\epsilon} d_n$$

This second relation has a clear physical interpretation. It is just a charge balance equation stating that the total amount of negative charges on the left must be equal to the total amount of positive charges on the right. Using the above two equations, we can solve for d_p and d_n as functions of ϕ_b , N_a , N_d , ϵ and e :

$$d_p = \sqrt{\frac{N_d}{N_a} \frac{1}{N_a + N_d} \frac{2\epsilon\phi_b}{e}} \text{ and } d_n = \sqrt{\frac{N_a}{N_d} \frac{1}{N_a + N_d} \frac{2\epsilon\phi_b}{e}}$$

The total size of the depletion layer is just given by $w = d_p + d_n$, so we find:

$$w = \sqrt{\frac{N_a + N_d}{N_a N_d} \frac{2\epsilon\phi_b}{e}}$$

For a GaAs junction with $\phi_b = 1$ V and $N_a = N_d = 10^{22} \text{ m}^{-3}$, we find $d_p = d_n = 2700 \text{ \AA}$ and $w = 5400 \text{ \AA}$.

3.3 pn junction under bias and I(V) curve

When one apply a potential difference V between the two ends of a pn junction one can easily realize that **forward bias** and **reverse bias** are not symmetrical. Let us investigate why. For this we will use two relationships linking the carrier densities at different points x that may easily be obtained from what we have seen before. Those relations are the following:

$$\text{For electrons, } n_c(T, x_1) = n_c(T, x_2) e^{e(\frac{\phi(x_1) - \phi(x_2)}{k_B T})}$$

and

$$\text{For holes, } p_v(T, x_1) = p_v(T, x_2) e^{-e(\frac{\phi(x_1) - \phi(x_2)}{k_B T})}$$

15) What is the relationship between the electron density at $x = -d_p$ and $x = d_n$ when no bias exists? Same question for the hole density.

solution:

Given that $p_v(T, x = -d_p) = N_a$ (the p-doped semiconductor is in its saturation regime to the left of the depletion region), $n_c(T, x = d_n) = N_d$ (the n-doped semiconductor is in its saturation regime to the right of the depletion region) and $\phi(d_n) - \phi(-d_p) = \phi_b$, it is easy to conclude from the formulas given above that:

$$n_c(T, x = -d_p) = N_d e^{-e \frac{\phi_b}{k_B T}} \text{ and } p_v(T, x = d_n) = N_a e^{-e \frac{\phi_b}{k_B T}}$$

16) When one applies a bias V, it tends to favor the flowing of charges under **forward bias** and unfavour it under **reverse bias**. So we will take V to be positive when it is a forward bias and negative when it is a reverse bias. With this convention how will the built-in potential ϕ_b change when an arbitrary bias V is applied? Explain what happens to the depletion region size when a forward

and reverse bias is applied to the junction and conclude about the asymmetry between forward and reverse bias.

solution:

When one applies a **forward bias**, the built-in potential is reduced by an amount V . On the contrary, applying a reverse bias means raising the potential barrier between the two ends of the depletion region. So in general, when one applies a bias V (positive for forward bias and negative for reverse bias) then the built-in potential becomes $\phi_b - V$. If we look at the expressions for the depletion region lengths, we find:

$$d_p(V) = \sqrt{\frac{N_d}{N_a} \frac{1}{N_a + N_d} \frac{2\epsilon(\phi_b - V)}{e}} \quad \text{and} \quad d_n(V) = \sqrt{\frac{N_a}{N_d} \frac{1}{N_a + N_d} \frac{2\epsilon(\phi_b - V)}{e}}$$

So the size of the depletion region is reduced when one applies a forward bias and increased when one applies a reverse bias. As a consequence, the flow of charges is eased when a forward bias is applied and almost prevented when a reverse bias is applied.

17) Using the above results, show that the change in **electron** density Δn_c at $x = -d_p$ between a configuration where no bias is applied and a configuration where a bias V is applied can be written as:

$$\Delta n_c = n_c^0 (e^{\frac{eV}{k_B T}} - 1)$$

Same question for the change in **hole** density Δp_v at $x = d_n$.

solution:

Using the results of questions 15) and 16), we find that:

$$\Delta n_c = N_d e^{-e \frac{\phi_b - V}{k_B T}} - N_d e^{-e \frac{\phi_b}{k_B T}} = (N_d e^{-e \frac{\phi_b}{k_B T}}) \left[e^{\frac{eV}{k_B T}} - 1 \right]$$

So we arrive at the required formula if we put $n_c^0 = N_d e^{-e \frac{\phi_b}{k_B T}}$. The same calculation for p_v gives us:

$$\Delta p_v = (N_a e^{-e \frac{\phi_b}{k_B T}}) \left[e^{\frac{eV}{k_B T}} - 1 \right] = p_v^0 \left[e^{\frac{eV}{k_B T}} - 1 \right]$$

Concluding remark: This change in the concentration of **minority carriers** on both side of the pn junction is responsible for a minority carrier diffusion current that behaves like:

$$J(V) = J_s (e^{\frac{eV}{k_B T}} - 1)$$

This current is responsible for the $I(V)$ response curve of a pn junction.