3.185 Test 2 (Amended)

Advanced Heat Transfer, Fluid Dynamics

Wednesday–Friday November 19–21, 2003

Welcome to the second 3.185 test of Fall 2003. Do not open this until you are told to begin.

- This is a 50-minute, in-class, closed-notes, closed book test, and you may not consult others during it, though you may use a calculator.
- You are expected to know and be able to use the equations on the review sheet. Other equations are given on the last page of this exam. If you need an equation which is neither on the review sheet nor on the equations page, ask and it will be written on the board.
- Feel free to use this test booklet as scratch paper, you can take it with you after the test, and you may ask for extra scratch paper or answer booklets at any time.
- Answer all of the questions, and be sure to show *all* work in your answer booklets, so if your numerical answer is incorrect, you might get partial credit for correct methodology and equations.
- Please begin your answers for each question on a new sheet of paper.
- You may answer the questions in any order.
- The test will be graded and scored, then returned to you along with a fresh test so you can correct it during the Thursday or Friday recitation. You may take as long as you like to correct the test, and if logistical difficulties prevent you from completing it in recitation, we will make arrangements for you to complete it at another time.
- Please indicate clearly in the Section space on the cover of your answer booklet when you will make your corrections, e.g. "Thursday recitation" or "Friday recitation".
- You may use any resources you like to help you to understand the material between the in-lecture portion and correction during recitation, including the instructor and TA (though we might not tell you exactly how to answer a question).
- Following the corrections, your test will be re-graded and scored. Your final score for the test will be the mean of these two scores.
- If you have questions, raise your hand where you are, and someone will come to answer them.

Knock it dead!

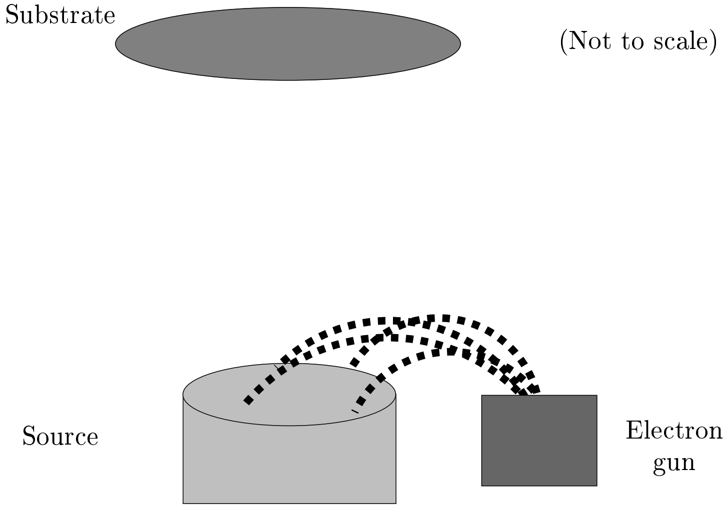
1. Write your name on all of your answer booklets (2 pts)

2. Aaliyah was: (pick one, 3 pts)
   - (a) President of Bosnia-Herzegovina during its civil war.
   - (b) A pop/R&B singer who lost her life at an early age in a plane crash.
   - (c) A leading NGO in the Saudi peace movement.
   - (d) Latin for "others", *e.g.* "*inter* ...", "*et* ..."

3. Molecular beam epitaxy deposition rate (15)

   In molecular beam epitaxy, depicted below, the material source is both heated by a low-power electron beam from above and water-cooled from below, resulting in sublimation from the solid state. The sublimated atoms arrive at the substrate at a low rate (flux), and with a low temperature (velocity), resulting in a high-quality film.

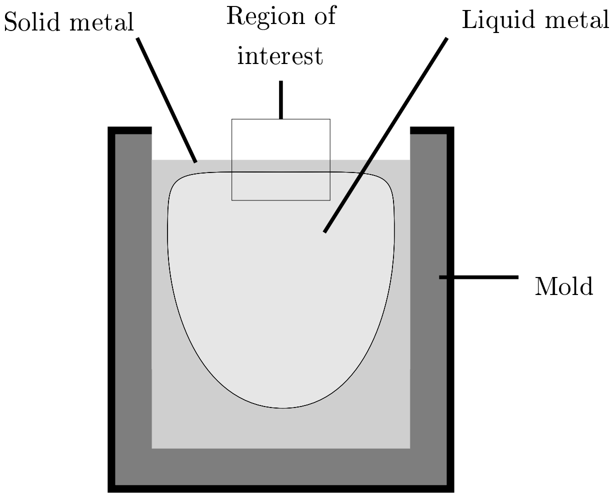
   The sublimation rate is easily low enough such that the evaporant atom directions follow a cosine distribution.



   - (a) Briefly outline the steps required to estimate the sublimation rate (that is, the flux J, *e.g.* in mol/cm^2/sec) based on the temperature at the source. (6)
   - (b) Write and expression for the net power which must be transferred to the solid source surface (electron beam power minus heat extracted by cooling water) as a function of J in order to maintain this sublimation flux. (4)
   - (c) Assume this is a "high vacuum" (*i.e.* extremely low pressure), so the atoms fly in a straight line to the substrate, like radiating photons. The source radius is 2 cm, the substrate radius is 9 cm, and the two are parallel, coaxial, and 30 cm apart. Estimate the fraction of sublimating atoms which arrive at the substrate, which is also the ratio of deposition flux at the substrate to sublimation flux at the source. (5)

4. Freezing by radiation and convection (39)

Castings with large open tops often "freeze off" by radiation and convection, forming a solid shell on top and trapping the liquid beneath it. (Because the liquid shrinks during solidification, this results in a large shrinkage cavity.) Here you will analyze the rate of solidification downward from the top surface in an low-carbon steel ingot casting due to these factors.



Assume that the liquid metal temperature is uniform, the environment absorbs all radiation (is "black"), and the top of the solid shell is grey. Also be careful not to confuse the two σs!

Iron data:

- Electrical conductivity near melting point: $\sigma = 5 \times 10^5 (\Omega \cdot \mathrm{m})^{-1}$.
- Density: $\rho = 7500 \frac{\mathrm{kg}}{\mathrm{m}^3}$.
- Heat capacity: $c_p = 500 \frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}}$
- Melting point: $T_m = 1800\mathrm{K}$.
- Heat of fusion: $\Delta H_f = 2.67 \times 10^5 \frac{\mathrm{J}}{\mathrm{kg}}$
- Radiative emissivity: $\epsilon = 0.6$.
- Heat transfer coefficient to air: $h = 100 \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}}$.

(a) Estimate the thermal conductivity of iron near its melting point. (5)

(b) Write an expression for the total radiative and convective heat flux from the top surface of the solidifying metal shell to the surrounding environment. (4)

(c) Assuming the environment is much colder than the shell ("0 Kelvin"), calculate a total "heat transfer coefficient" which is the ratio between heat flux and absolute temperature. (6)

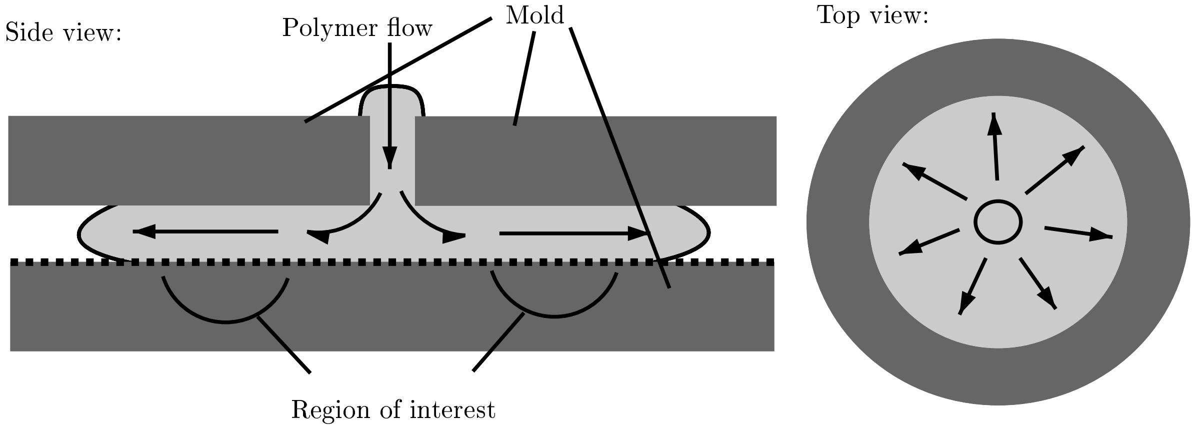
(d) Use your heat transfer coefficient from part 4c to estimate the thickness of solid metal Y at which temperature can no longer be considered uniform (where the Biot number reaches 0.1). (9)

(e) Estimate the rate of growth of the solid while solidification rate is limited by radiation/convection from the top surface (that is, while solid temperature can be considered uniform). Assume quasi-steady-state behavior in the solid, meaning that the flux is the same throughout the thickness. (9)

(f) Set up the equation for solidification rate limited by both radiation/convection from the top and also (quasi-steady-state) conduction through the solid. (But don't solve it — unless you like using Newton's method on a test! :-) (6)

5. Injection molding a CD (17)

   CDs are made cheaply in large quantity by injection molding polycarbonate into a nickel mold with precisely-machined bumps which form the ones and zeroes. We would like to estimate the shear stress exerted by the polycarbonate on the nickel, in order to assess the lifetime of the mold bumps (i.e. number of CDs it can make accurately).



   The polymer flows from the center outward, and the viscosity is high enough and the mold thin enough (~1mm) that Reynolds number is very small. (Among other things, this means that the "entrance length" is even smaller than the ~1cm circle which is cut out of the center.)

   (a) Using the accompanying handout *Solving Fluid Dynamics Problems*, state the assumptions which you can make about flow through the thin mold. You can ignore the outer region near the advancing front (the "leading edge" of the polymer), and also ignore the region very close to the center. Also, assume that the flow rate is constant (steady-state), and that the polycarbonate behaves like a Newtonian fluid here. (8)

   (b) Cancel the appropriate terms on the cylindrical Navier-Stokes equations, which you will detach from the handout and turn in with your test answers. You do *not* need to (and may not be able to) reduce it to a simple system which you can easily solve. (9)

6. Flow rate and pressure in a tube (24)

   Water data:

   - Density: $\rho = 1 \frac{\text{g}}{\text{cm}^3}$.
   - Viscosity: $\mu = 0.01 \frac{\text{g}}{\text{cm}\cdot\text{s}}$.

   (a) Briefly outline the steps required to calculate the pressure drop ΔP required to pump a Newtonian fluid through a horizontal tube at a given flow rate Q (assuming fully-developed axisymmetric flow, uniform density and viscosity, etc.). (8)

   (b) For water flowing through a 1 cm diameter smooth tube, calculate the pressure drop per unit length $\Delta P/L$ required to pump water at flow rates of: $0.25\pi \frac{\text{cm}^3}{\text{sec}}$, $2.5\pi \frac{\text{cm}^3}{\text{sec}}$, $25\pi \frac{\text{cm}^3}{\text{sec}}$. (12)

   (c) Why is the result at $Q = 25\pi \frac{\text{cm}^3}{\text{sec}}$ qualitatively different from the other two? (4)

Equation sheet

- Radiation viewfactors between parallel opposed discs (W^{3}R p. 398):
- Sublimation heat flux (ΔH_s is sublimation enthalpy in J/mol): $q_V - q_S = \Delta H_s \cdot J$.
- Wiedmann-Franz constant: $L = 2.45 \times 10^{-8} \frac{\mathrm{W\Omega}}{\mathrm{K}^2}$, radiation constant: $\sigma = 5.67 \times 10^{-8} \frac{\mathrm{W}}{\mathrm{m}^2\mathrm{K}^4}$.
- Convective heat transfer boundary condition: $q = h(T_s - T_{env})$.
- Heat conduction Biot number: $\mathrm{Bi} = hL/k$.
- Melting/solidification heat flux (ΔH_f is enthalpy of fusion in J/kg):

$$q_L - q_S = \rho \Delta H_f \frac{dY}{dt}.$$

- Hägen-Poisseuille equation for laminar flow of a Newtonian fluid through a tube:

$$Q = \frac{\pi \Delta P R^4}{8 \mu L}.$$

- Flow rate and average velocity (A_{xs} is cross-section area):

$$Q = u_{av} A_{xs}.$$

- Friction factor vs. Re for flow in a tube (W^{3}R p. 188):