3.185 Math Quiz

Wednesday October 1, 2003

This is an in-class, closed-book, closed-notes quiz. You may use a calculator, and use this sheet as scratch paper. If you need more scratch paper, ask and it will be provided.

1. Write your name on the top of all answer booklets you turn in. (5)

2. Vector algebra (25)

   For the two vectors: $\vec{A} = (1, 2, 3)$, $\vec{B} = (4, 5, 6)$

   (a) Calculate their dot product $\vec{A} \cdot \vec{B}$. (7)
   (b) Calculate their cross product $\vec{A} \times \vec{B}$. (9)
   (c) Calculate their outer product $\vec{A}\vec{B}$ (also written $\vec{A} \otimes \vec{B}$). (9)

3. Vector calculus (50)

   A rod of radius 1 centered on the z-axis is rotating in a fluid at an angular velocity of 1. In the fluid, this leads to the velocity field:
   $$\vec{u} = \frac{1}{r}\hat{\theta}, \text{ so } u_r = u_z = 0, u_\theta = \frac{1}{r}$$
   or in cartesian coordinates:
   $$\vec{u} = \frac{x\hat{y} - y\hat{x}}{x^2 + y^2}$$

   (a) Sketch the vector field in the first quadrant for r between 1 and 3 (that is, for x and y between 0 and 3, but not inside the cylinder). (15)
   (b) Show that the θ-component of velocity satisfies the differential equation: (10)
   $$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru_\theta)\right) = 0.$$

   (c) What is the curl of this vector field, $\nabla \times \vec{u}$? (It's z-component, cartesian coordinates.) (25)

4. Error function derivatives (20)

   For the error function defined by
   $$\text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-\xi^2}d\xi,$$
   calculate:
   $$\frac{d}{dy}\text{erf}(y^2)$$
   and simplify as much as possible.