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SAL: Hi. I'm Sal. Today, we're going to solve problem nine of fall 2009, the final for 3091.

The problem reads, the phase diagram of the binary system neodymium, praseodymium and Gd is given below. There are two allotropes-- alpha, which is hexagonal close pack-- HCP-- and beta, which is body center cubic-- BCC.

Now this problem obviously is about phase diagrams. So before you attempt to solve the problem, you need to make sure that you understand how to read phase diagrams and also how to apply the lever rule and know where in the phase diagram to apply the lever rule.

So if we look at our phase diagram, which I have drawn over here for part A-- the part A asks, explain why a lenticular phase diagram is to be expected for the binary system.

So this is a lenticular phase diagram and a lot of students when answering this problem get confused and say, oh, because the components are soluble together, that's why you get a lenticular. That's correct, but that information you can get from the phase diagram. It's obvious by looking that you have an alpha phase that you have from zero to a 100% that can be miscible together, but it doesn't really explain the answer.

Now the best thing to do is to look at the periodic table of elements and look at the physical and chemical properties of the elements. So here we're comparing to-- we're comparing neodymium and praseodymium, which are two components that are making the alloy. So here this is atomic percent praseodymium on your x axis and on your y axis, you have temperature in degrees Celsius. So at zero percent praseodymium, we have a 100% neodymium and at 100% praseodymium, we have zero percent neodymium. That pretty much comes from the phase diagram.

So why would these two have formed a lenticular phase diagram? Well, if you look at the periodic table, you'll notice that these two elements lie right next to each other. So what does that mean? That means that chemically, they're pretty much almost the same.

To answer this question for part A, I would look at it and I would notice that actually both these elements have the same crystal structure. So I would write down, they have the same crystal structure. They also have pretty much the same electronegativity or very similar electronegativity. So very similar electronegativity. Also, they have pretty

much the same atomic radii--atomic radius. And also very similar, very close to each other-- the same density. So the density is almost the same. So because these two components have the same properties together-- the same size, same crystal structure, then you should expect that they should form a miscible lenticular phase diagram, which is what we have here. So if you were to answer that for part A, that would give you pretty much all the points and that hints that you understand how to look at the chemical properties of the elements and the physical properties of the elements and compare them together and rationalize why the phase diagram looks the way it does.

Part B asks-- I'll come over here to do part B. At each point, I identify all the phases present in equilibrium. II-- state the composition of each phase and III-- calculate the relative amount of all phase presence. And it asks us to analyze the three points that we have on our phase diagram. So we have one for I, II, III--

So number one-- if we look at our phase diagram, we lie exactly where your alpha phase is. So to answer the first part of the problem is identify all the phases present-- we only have the alpha phase here. So if you were to write down alpha phase, that's correct.

The part two says, state the composition of each phase. So what's the composition of the alpha phase in here? Well, if I trace down my line down to my x axis, I notice that for the atomic percent of praseodymium, pretty close to about 18%. So it's 18 atomic percent here. So to answer the question for part I, I would notice that-- I would write down that it's alpha phase.

For the second part, we have 18%-- 18 atomic percent praseodymium, which leaves behind about 82% atomic percent of neodymium.

And for part three-- part three asks about the relative amounts of the phases present-- we only have the alpha phase in that part so I would say that we have 100% pure alpha, which is-- if you write this, that's correct.

Now for the second point that we're going to analyze in our phase diagram number two-- if we come over here and look at it, you'll notice that now we lie in between or between two phases so where the lenticular region is at. So here, I don't have pure beta or pure liquid. I have a combination of both beta and liquid and this is exactly where you need to apply the lever rule. You can't apply the lever rule here because it's just 100% pure phase. You can only apply when you're in a region where you have two phases that coexist in equilibrium.

So if I go ahead and analyze part two-- part I-- the phases that are present are beta and the liquid phase.

For part II, it asks, what are the compositions? Well, if I blow up my lenticular phase diagram-- so I'm sitting right here and this is sitting at around 62 two atomic percent of Pr-- praseodymium and it asks about the phases or the relative composition of each phase. Well, over here, I have beta, over here I have the liquid phase and in between

I have the beta plus the liquid that are coexisting in equilibrium. So if I want to know how much-- what's the composition of my beta phase that I have here-- well, I'm going to have to go to the line where both intersect with the beta and the beta plus liquid intersects. And if I analyze this point-- if I look down what the composition of that is for that line, I see that I have about 56 atomic percent of praseodymium. So to start answering part two, I would say that if I look at beta-- in the beta phase, I have 56% or atomic percent of Pr and if I have 56% of Pr, then this leaves just 44 atomic percent of neodymium and if I look at my liquid-- so I'm going to just move this over here. I look at the liquid, which is going to be where it intersects in the liquid region and if I trace it down, I see that this is about 70% or so. So here I have around roughly 70%. If you trace it down to your x axis, this is 70% or 70 atomic percent Pr, which leaves 30 atomic percent of neodymium. So if you were to write this down for the liquid, just write an arrow here so I won't have to rewrite that-- that would be the answer for part two because again, we're analyzing this point, which is point number two and we're in between two phases. So that covers the second part of part B.

And for the third part-- I'll go ahead and I'll put up here. That one now is where we have to apply the lever rule because now we have to figure out exactly how much of each phase is present at equilibrium. So if I look at my diagram, it's going to be very simple. If I want to know how much beta I have in this mixture, you're simply going to apply the lever rule and for beta, it's going to be whatever this magnitude is or whatever the difference between these two points are over the length of the whole existence at that temperature of the length of the whole composition that you have. So if I do the simple calculation for beta-- beta is going to be simply  $70 - 62$  over  $70 - 56$  and this essentially just equals  $8$  over  $14$ , which is about 57 atomic percent praseodymium. So-- sorry-- not percent Pr-- percent beta phase. So in beta, I have 57%. So that means that for my liquid-- I can do two things for this problem, for the liquid. I know that everything is normalized to 100%. So the difference between 157 will be the liquid composition where I can go ahead and apply the lever rule again and do the math. So if I apply the lever rule again for my liquid phase, it's going to simply be  $62 - 56$ . We're coming back on our-- looking at the liquid phase that is coexisting in this equilibrium at this temperature and I want to know how much liquid-- it's going to be the magnitude of this over the total length of my composition there-- that's what I'm doing here. Again, this is  $70 - 56$ . So this would end up being just  $6$  over  $14$ , which is essentially just-- if you have 57% here, if you do the math, you're going to get 43% of liquid. So part three-- when you apply the lever rule, you see that at this point you have more beta than you have liquid-- and it kind of makes sense because if you look at your diagram, your point is sitting closer to the beta phase. So I would expect intuitively that I would have more beta present than the liquid present and that's exactly what the lever rule tells us. So that's point number two for part B.

And if I look at point number three now, which I'll write right here-- this is essentially the same thing as point number one. But now point number three-- we're no longer lying in alpha. We're lying in the beta phase. So if I

trace down what my composition is here, I'm about 78% of Pr-- so to answer the first part I-- beta-- the second part II, which is dealing with the compositions of that beta phase is going to be 78% or 78 atomic percent Pr and that leaves just 22 atomic percent neodymium-- and for the third part III-- what is the relative amounts of the phases present? Not the composition, the phases. I'm exactly where only pure beta exists so I can say that I have 100% beta. So that pretty much does it for this problem. Again, knowing how to read phase diagrams, knowing how to apply the lever rule and where to apply the lever rule will allow you to very easily solve the problem. A lot of students make the mistake of trying to apply the lever rule when you have a pure phase. I mean, that makes no sense because said lever rule pretty much calculates what fraction of the phases are present in that equilibrium mixture. So keep in mind that you can only do that when you have a region in your phase diagram, when you have two phases that are coexistent in equilibrium.