SAL BARRIGA: Hi, I'm Sal. Today, we're going to solve Problem #5 of Exam 3 of Fall 2009. Now before you start this problem, there are a couple of things that you need to know in order for you to fully understand the science behind it.

And so what you need to know is Fick's first law. Not just knowledge of it, but how it works. Also, Fick's second law. And not necessarily the equation form, but the solution to Fick's second law, which is a function of concentrations, and which we will refer to to solve the problem. And also know popular dislocations, how they help your material and whatnot.

So the problem reads as follows. Specimens of steel are being surface hardened by the introduction of carbon. The concentration of carbon at the free surface of the steel is kept constant at c sub s. Chemical analysis of a number of specimens indicate that after a time-- t sub 1-- the carbon concentration has the value of c star at a depth from the surface-- x1-- shown below, which is shown on your graph. So this is given to you.

So this pretty much is your concentration profile as a function of position. And it's telling you what's happening. So a good thing to do is draw a picture of the science that's happening here.

And they tell you, first of all, I'm going to draw it underneath here that you have a rod of steel. And so this is steel. And I have a certain fixed concentration here at the surface of carbon. So I have some type of a carbon flux in my surface. And this is a visual image of how it looks.

And what the problem says is that you have a certain concentration on your surface, which is given by c sub s. And then it starts telling you that chemical analysis was able to go ahead and measure this concentration at two different values inside your position at different times. So if I continue reading the problem, it says that chemical analysis of a number of specimens indicate that after a time-- t1-- the carbon concentration has a value of c star.

So this one right here-- this is x1-- so this has a value of x1 at t1. And it also tells you that after a certain time-- t2, which is equal to 2 times t1-- the carbon concentration has a value of c star, so the same value at a depth of x2. So now, this thing says that at a depth of x2, at a time of t2, we measure the same concentration, c sub star in the specimen. And it tells you that x2 equals 2x1, and it also tells us that t2 equals 2t1. Now they tell us that for a reason, because it's probably expected for us to use this to solve the problem, so don't forget that.

So the question is, under these conditions, would you describe the rate of carburization as one, steady state diffusion. Two, transient state diffusion. Or three, not governed by any diffusion, i.e., rate limited by some other process. Now you have to justify your choice. So this is pretty much a problem of process of elimination.

And for part a, which is-- I'll label this part a-- it wants us to do three things. So the first thing, it asks us if it's steady state. So the first thing to ask yourself is, well, what's steady state? And the definition of steady state is that whatever phenomenon is happening doesn't vary with time. But the problem already tells you that it varies with time. It tells you that at a certain time you measure this and at another time you measure this, and you have this flux coming in, and this is what's happening.

So from just reading the problem, I can already say that this process can't be steady state, because your concentration profile varies as a function of time. So for one, I would write, can't be steady state due to the concentration being a function of time. So one is out, so we can scratch that. It's not a steady state.

Now two. This is where it gets a little bit more complicated. Two is a transient state diffusion.

Well, how do we know whether or not this is governed with time? Well, you would argue, because this is not steady state, then it has to be time dependent. But, there are certain rules that you need to know to be able to make that justification. First of all, if it's time dependent, then whatever your concentration and whatever your position in your time is, has to fit into your solution to Fick's law.

So if I look at my solution to Fick's law right here, I know that on the left side of my equation, I have the concentration at any point in x-- that's what c stands for-- minus the concentration at my surface, which is given as cs-- sorry, this is supposed to be sub s-- divided by the initial concentration. So c sub naught is if there was already some carbon in my steel, minus, again, the surface concentration. This equals to an error function-- which is the solution to Fick's second law-- and this error function has the values of position, diffusion coefficient, and time.

So in order to verify if it's a transient state, we're going to go ahead and take that equation and break it down. So if I assume that there's no carbon, I can go ahead and write this down. Assume c0 is 0, just to make the equation easy. I can, because there was no information given about that. I can go ahead and-- because we're trying to figure out if it's a transient state-- write this down.

So c minus c sub s over negative c sub s. This equals to my error function, which is x divided by 2 squared of Dt. And I know that at x1 and time-- t1-- I have this value of my concentration, c sub star. So I'm going to go ahead and plug that in here into this equation. So I know that my concentration is c sub star.

So that's what we measured, c sub star minus c sub s over negative c sub s. This equals to my error function, x1

over 2 square root of Dt1. So this is for the first time point, x1, t1.

Now I want to go ahead and do the same thing for the second point. And if I do it for the second point, I know that just by looking at the equation, I have the same concentration at x2, and I have the same surface concentration. So what that tells me is that this value is going to be the same for x1, t1, and x2, t2.

So if I go ahead and plug this into my equation. I can go ahead and look at this and say, this is for t equals 1 for that, and this is going to be-- or, t equals t sub 1, sorry-- and this is t sub 2. I have c star minus cs all over negative cs. This equals to my error function of x2 over 2 square root of Dt2.

So I know that this value is the same as this value. So therefore, this should be the same as this. The argument in both your error functions. So we have to go ahead and prove that.

Well, how are we going to do that? Do I know x2 as a function of x1? I do, because that's given in the data. And I know again, that x2 equals 2x1 and t2 equals 2t1.

So what I want to know, does x1 over 2 root Dt1, does this equal to x2 over 2 root Dt2? Well, I'm going to go ahead and plug in my values that I know, and this becomes 2x1 divided by 2 times the square root of D-- and t2 happens to be 2t1. So those two cancel each other out, and I end up getting that this is actually equal to x1 over square root of 2Dt1.

Now this and this are not the same. Because this has a square root of 2, where the other one is just a 2. So because it did not satisfy the solution to Fick's second law, we can say that it's not case two either. This is not transient state diffusion.

So therefore, by the process of elimination, it has to be governed by some other phenomena. And that is the answer to this problem. The fact that it's governed by something that we don't have enough information to answer. We don't have enough data points to answer. So that's part a, which is good.

Now part b asks, if the steel specimens in part a were replaced with steel specimens of identical composition but with a dislocation density 10 times greater than that of the steel in part a, how would the rate of uptake of carbon change? Explain. Again, this requires you to know properties of dislocations, and what dislocations do to your materials. Your metals, for example.

What is a dislocation? Well, a dislocation is-- if you have a bunch of a plane of atoms-- what a line dislocation is, or an edge dislocation is, this line defect such that it just terminates. So you have a plane of atoms that just comes to a point and then it stops.

And what do you create here? Well it creates void, and void facilitates diffusion. It increases-- in our case, we're talking about carbon diffusion into steel-- so I would argue that the more dislocations that you have, the greater the diffusion should be. Because you have more void-- more spaces-- for your carbon atom to jump into and from.

So for part b, just by arguing the fact that dislocations represent defects in your crystal-- specifically a defect that can create void-- then with that argument, the fact that more void means higher diffusion, I would conclude that-- for this part-- that my rate of diffusion of carbon should increase when using steel of a greater dislocation density. So the rate of carbon uptake or diffusion of carbon into your steel increases as you increase your dislocation density. And with that, you're to be able to solve a lot of these problems. And these are common in 3.091. And again, knowledge of Fick's first and second law, how to use the solutions to Fick's first, second law, and knowing what a dislocation is, that will help you a lot in solving these problems.