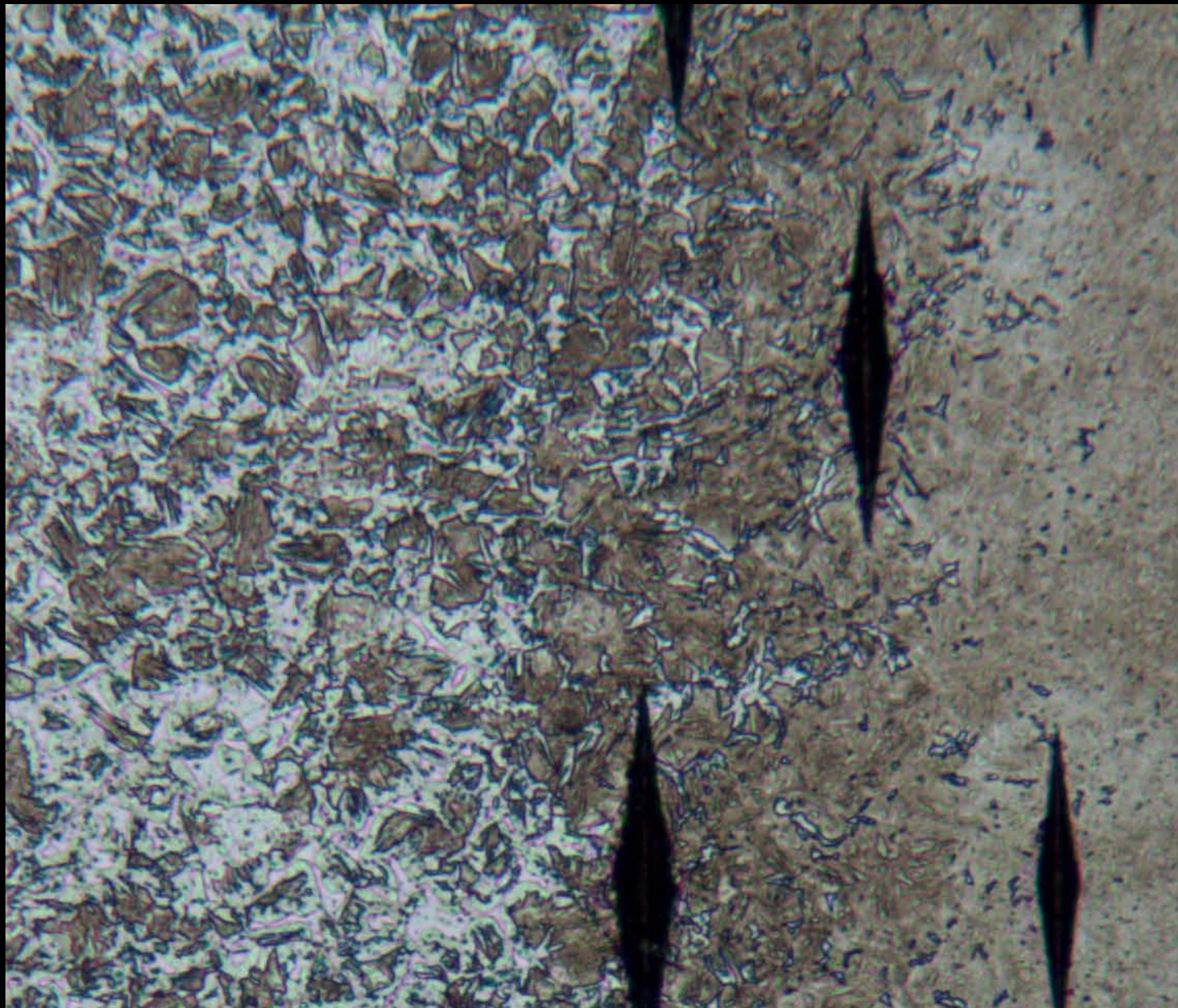


That Fick's 2nd Law problem from class today.



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high-
carbon
steel

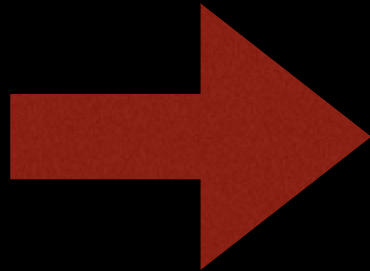


low-
carbon
steel

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Question: if the low carbon steel has a carbon content of 0.25 wt% and I have a source of 1.2 wt% carbon at the surface, how long should I expose the surface at $T=900^{\circ}\text{C}$ such that at 0.5 mm into the surface the carbon content is 0.8 wt%?

at surface,
concentration is
 $C_s = 1.2 \text{ wt } \%$



$x=0 \text{ mm}$
have $C_s = 1.2 \text{ wt } \%$

$x=0.5 \text{ mm}$

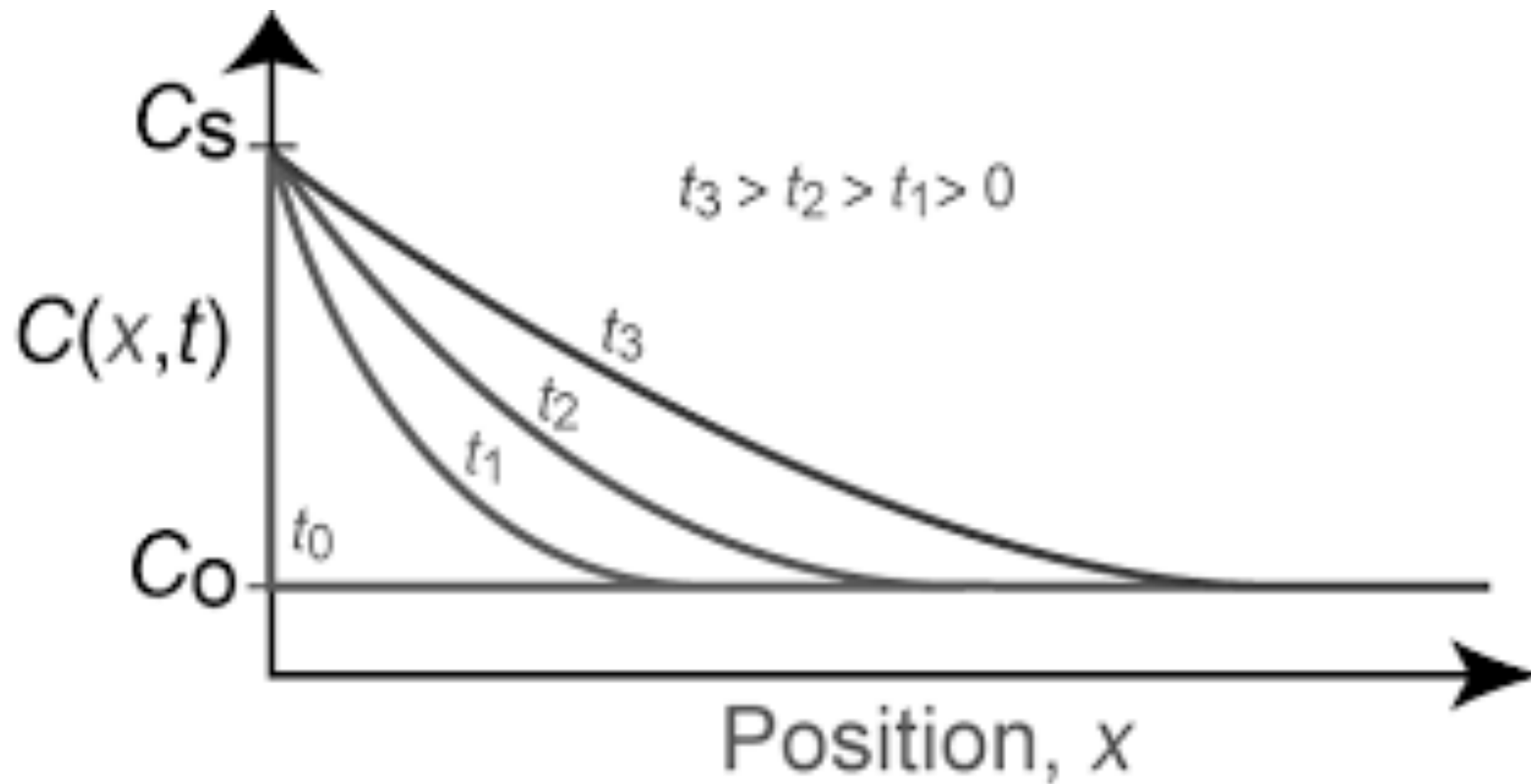
want
 $C=0.8$
wt% here

low-
carbon
steel
(started
with
 $C=0.25$
wt %)

D for carbon in
iron at this
temperature
 $= 1.6 \times 10^{-11} \text{ m}^2/\text{s}$

$C_o = 0.25$
wt% out
here

at $x=0.5 \text{ mm}$ in from the surface, we want $C=0.8 \text{ wt } \%$



Non Steady-State Diffusion $C=C(x,t)$

Fick's Second Law

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$$

$$\frac{C(x,t) - C_o}{C_s - C_o} = 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

Initially uniform carbon concentration = 0.25 wt%

Concentration @ surface = 1.2 wt%

T=950 Celsius.

How much time is needed to get a carbon content of 0.80 wt% at a position 0.5 mm below the surface?

D for carbon in iron at this temperature = $1.6 \times 10^{-11} \text{ m}^2/\text{s}$

$$\frac{C_x - C_o}{C_s - C_o} = \frac{0.80 - 0.25}{1.20 - 0.25} = 1 - \operatorname{erf} \frac{(5 \times 10^{-4} \text{ m})}{2\sqrt{(1.6 \times 10^{-11} \text{ m}^2/\text{s})(t)}}$$

can use table or can use erf trick: if $z \leq 0.65$ then $\operatorname{erf}(z) \sim z$
getting the erf() term onto one side, we have:

$$0.42 = \operatorname{erf}() \longrightarrow 0.0005 / [2 * \sqrt{(1.6 \times 10^{-11} * t)}] = 0.42$$

solve for t $\longrightarrow t \sim 7$ hours

TABLE 5.1 Tabulation of Error Function Values

z	$erf(z)$	z	$erf(z)$	z	$erf(z)$
0	0	0.55	0.5633	1.3	0.9340
0.025	0.0282	0.60	0.6039	1.4	0.9523
0.05	0.0564	0.65	0.6420	1.5	0.9661
0.10	0.1125	0.70	0.6778	1.6	0.9763
0.15	0.1680	0.75	0.7112	1.7	0.9838
0.20	0.2227	0.80	0.7421	1.8	0.9891
0.25	0.2763	0.85	0.7707	1.9	0.9928
0.30	0.3286	0.90	0.7970	2.0	0.9953
0.35	0.3794	0.95	0.8209	2.2	0.9981
0.40	0.4284	1.0	0.8427	2.4	0.9993
0.45	0.4755	1.1	0.8802	2.6	0.9998
0.50	0.5205	1.2	0.9103	2.8	0.9999

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