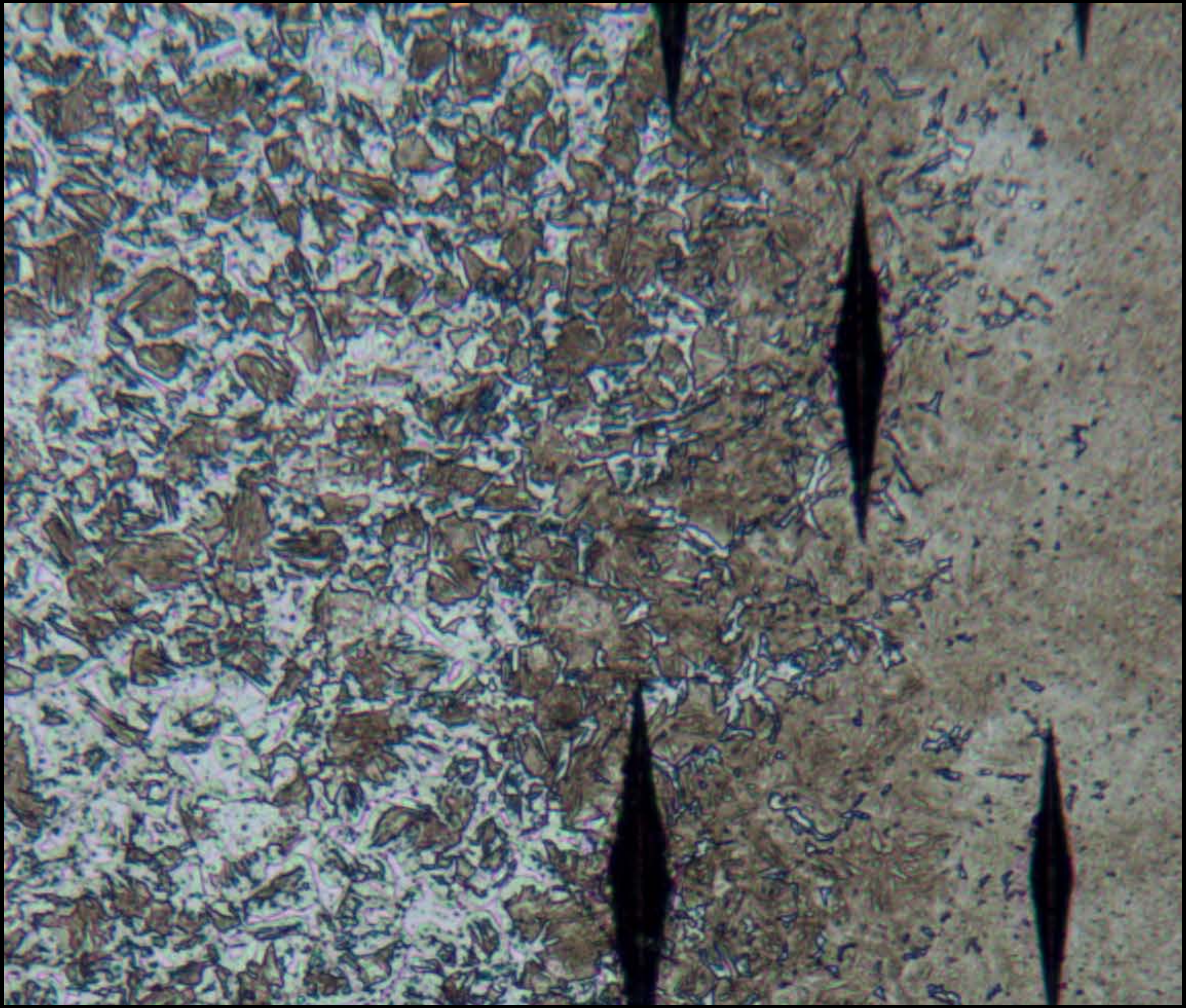
That Fick's 2nd Law problem from class today.

high-carbon steel



low-carbon steel

Question: if the low carbon steel has a carbon content of 0.25 wt% and I have a source of 1.2 wt% carbon at the surface, how long should I expose the surface at T=900^{o}C such that at 0.5 mm into the surface the carbon content is 0.8 wt%?

at surface, concentration is Cs = 1.2 wt %

x=0 mm
have C_s=1.2 wt %

D for carbon in iron at this temperature =1.6 x 10^{-11} m^2/s

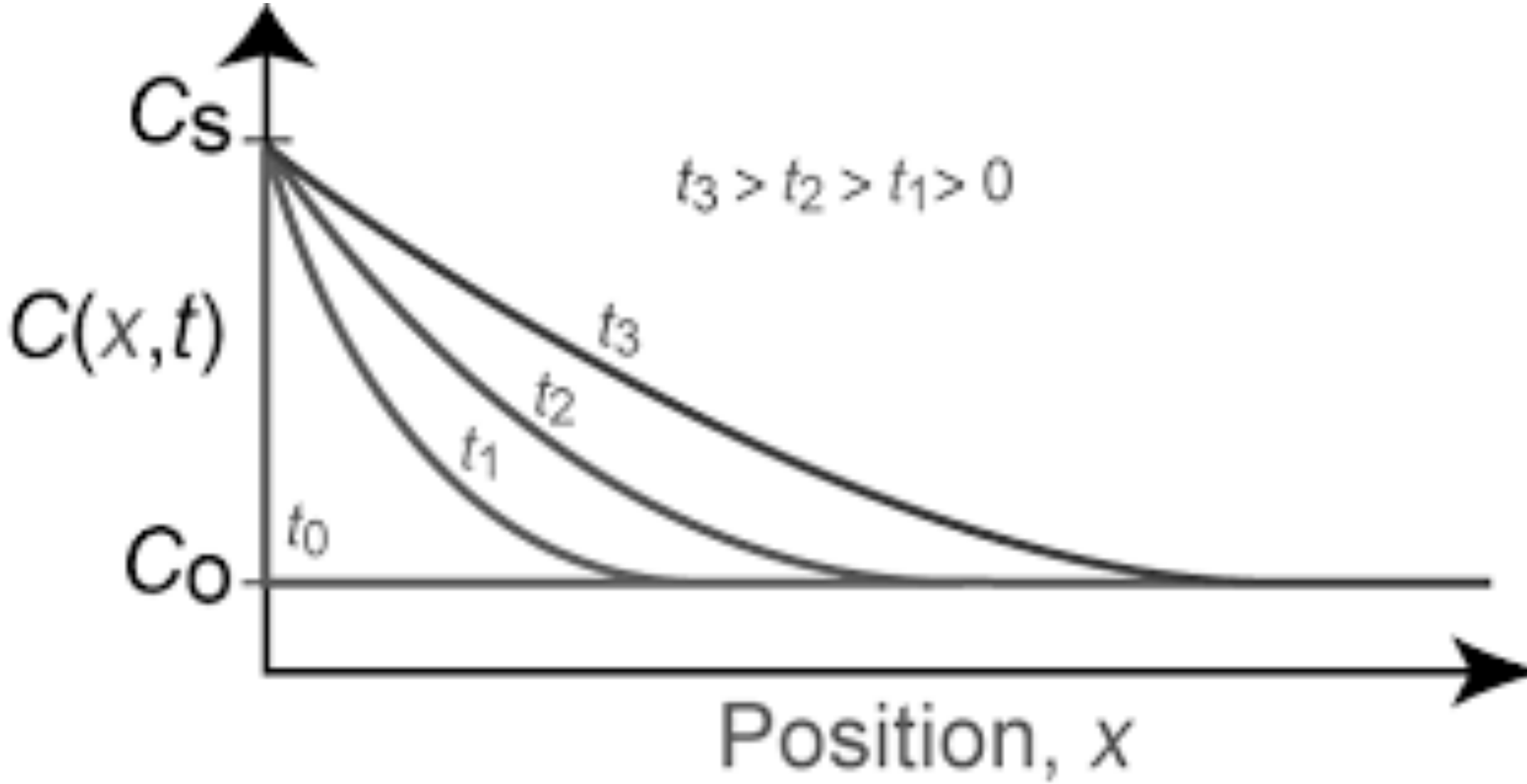
x=0.5 mm

want C=0.8 wt% here

low-carbon steel (started with C=0.25 wt %)

C_o=0.25 wt% out here

at x=0.5 mm in from the surface, we want C=0.8 wt%



Non Steady-State Diffusion C=C(x,t)

Fick's Second Law $$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}$$

$$\frac{C(x,t)-C_o}{C_s - C_o} = 1 - \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

Initially uniform carbon concentration = 0.25 wt%
Concentration @ surface = 1.2 wt%
T=950 Celsius.

How much time is needed to get a carbon content of 0.80 wt% at a position 0.5 mm below the surface?

D for carbon in iron at this temperature =1.6 x 10^{-11} m^2/s

$$\frac{C_x - C_o}{C_s - C_o} = \frac{0.80 - 0.25}{1.20 - 0.25} = 1 - erf \frac{(5 \times 10^{-4} m)}{2\sqrt{(1.6 \times 10^{-11} m^2/s)(t)}}$$

can use table or can use erf trick: if z<=0.65 then erf(z) ~z
getting the erf() term onto one side, we have:
0.42 = erf() —> 0.0005/[2*sqrt(1.6 x 10^{-11}*t)] = 0.42
solve for t —> t=~7 hours

TABLE 5.1 Tabulation of Error Function Values

z	$erf(z)$	z	$erf(z)$	z	$erf(z)$
0	0	0.55	0.5633	1.3	0.9340
0.025	0.0282	0.60	0.6039	1.4	0.9523
0.05	0.0564	0.65	0.6420	1.5	0.9661
0.10	0.1125	0.70	0.6778	1.6	0.9763
0.15	0.1680	0.75	0.7112	1.7	0.9838
0.20	0.2227	0.80	0.7421	1.8	0.9891
0.25	0.2763	0.85	0.7707	1.9	0.9928
0.30	0.3286	0.90	0.7970	2.0	0.9953
0.35	0.3794	0.95	0.8209	2.2	0.9981
0.40	0.4284	1.0	0.8427	2.4	0.9993
0.45	0.4755	1.1	0.8802	2.6	0.9998
0.50	0.5205	1.2	0.9103	2.8	0.9999